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ABSTRACT

For classes using the Comprehensive School Mathematics Program (CSMP) for the first time in the third grade, the schedule of lessons is modified to include the nineteen introductory lessons described in this supplement. These lessons acquaint the students with the minicomputer, the languages of strings and arrows, and with negative numbers, thus providing the necessary background for lessons in the third-grade program. The notes to the teacher include: how to use the third-grade entry schedule, materials, and a note on the subtractional algorithm. (Author/MK)

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CSMP Mathematics for the Upper Primary Grades

Comprehensive School Mathematics Program

CEMREL, Inc.

Final Experimental Version

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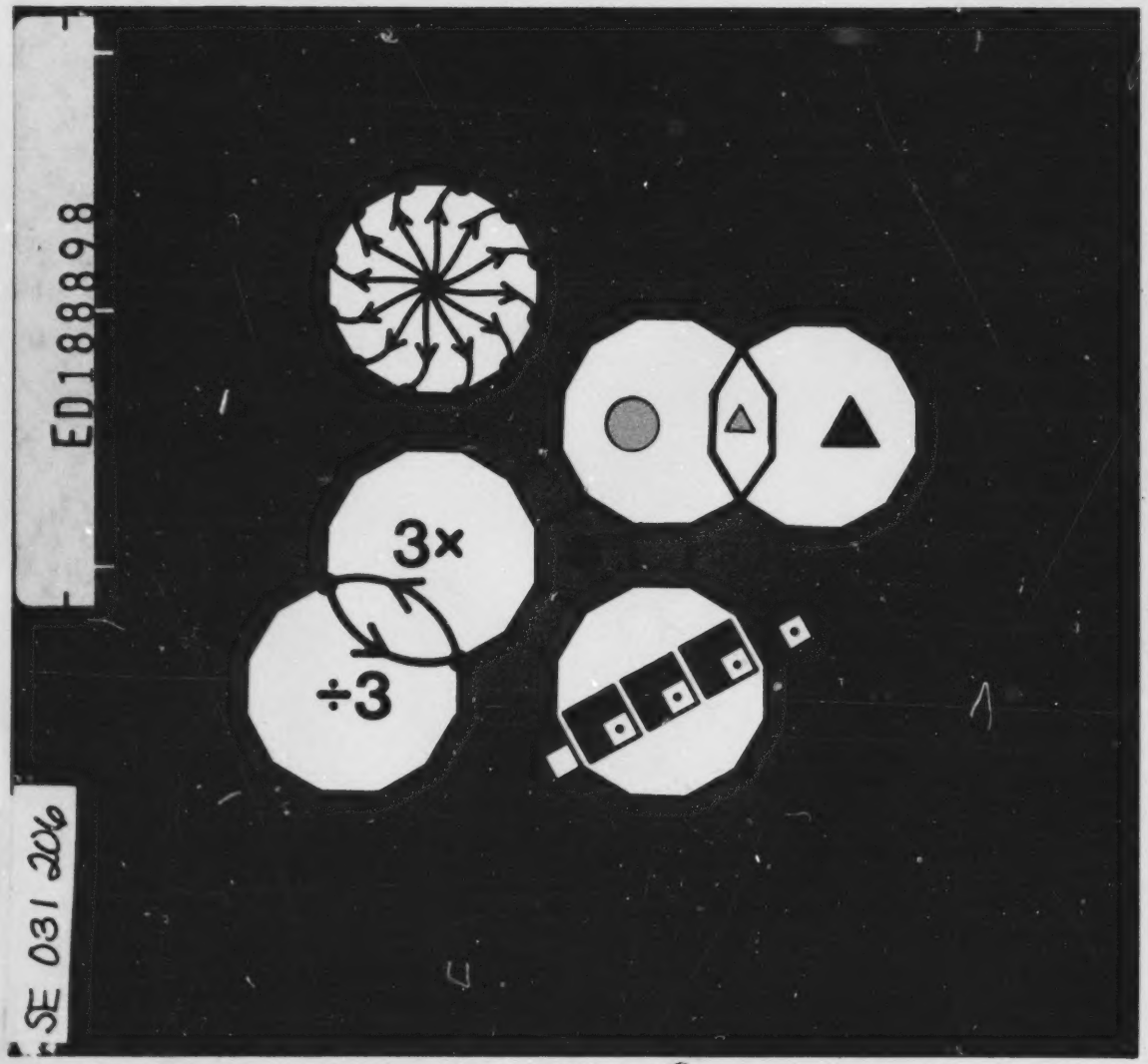
Teacher's Guide

A SUPPLEMENT FOR THIRD GRADE ENTRY CLASSES



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Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

A SUPPLEMENT FOR THIRD GRADE ENTRY CLASSES

CEMREL, Inc.

MAY 1 6 1980

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During the 1978-79 school year, Marilyn Levinson and Betty Cosgrove tried out the Third Grade Entry Program with their classes at Flynn Park Elementary School in University City, Missouri. We are grateful to those teachers for their cooperation, insights, and enthusiasm. Their observations and their students' responses helped to decide which lessons to include in this supplement.

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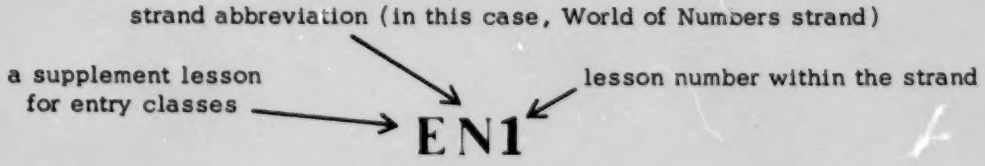
THE THIRD GRADE ENTRY PROGRAM

The veteran third grade program is described lesson by lesson in the volumes entitled Mathematics for the Upper Primary Grades, Parts III and IV (UPG-III and UPG-IV). For classes using the CSMP Program for the first time in the third grade, we have modified the schedule of lessons from UPG-III and IV to include nineteen introductory lessons. These nineteen lessons acquaint the students with the Minicomputer, with the languages of strings and arrows, and with negative numbers, thus providing the necessary background for lessons in the third grade program. This booklet, A Supplement for Third Grade Entry Classes, contains the descriptions of the nineteen introductory lessons. A two-semester schedule for the third grade entry program appears as a separate item in the classroom set of materials.

HOW TO USE THE THIRD GRADE ENTRY SCHEDULE

As a teacher of a third grade entry class, you will be teaching lessons from both this booklet and from the UPG-III and IV guides. When referring to the schedule, pay close attention to the letters before the lesson numbers. An "E" before a lesson number indicates that its description can be found in this booklet. For example, Lesson EN1 is the first supplement lesson to the World of Numbers strand; its description begins on page 1 of this booklet.


strand abbreviation (in this case, World of Numbers strand)
a supplement lesson for entry classes
lesson number within the strand
EN1

If there is not an "E" before a lesson number, then the description of that lesson can be found either in the UPG-III guide or in the UPG-IV guide. In the third grade entry schedule, lessons listed below the heavy black line on the second semester are described in the UPG-IV guide with the exception of Lesson EL6 which is described in this booklet.

MATERIALS

In addition to the materials needed for UPG-III and IV, you will need these materials for the Third Entry Program:

- Language Poster #2 for UPG-II
- "temperature data and chart" sheets
- thirty copies of the workbook FISHING FOR NUMBERS, Part II
- an outdoor celcius thermometer (not provided in your classroom set of materials)

There are some materials in your classroom set you will not need because the lessons requiring those materials are not scheduled for entry classes.

Most of the worksheets that your class will need for the Third Grade Entry Program can be found in the students' worksheet booklet for UPG-III and IV. There are two lessons described in this entry supplement, EN1 and EW1, that have worksheets. Because EN1 is similar to N1 in UPG-III, the latter is not included in the Third Grade Entry Program although its worksheets are to be used with Lesson EN1. You will find a note in Lesson EN1 reminding you of which worksheets to use. The two worksheets for EW1 are pictured on pages 107 and 108 of this supplement, but classroom quantities are not provided for your students. These are the only two worksheets that you will need to reproduce.

A NOTE ON THE SUBTRACTION ALGORITHM

In the CSMP Program your students will be introduced to a subtraction algorithm, "Nick's method", but not until the second semester of third grade. However, there will be many earlier lessons devoted to finding ways to do subtraction calculations (pre-algorithm experiences). Your students might come to the third grade having seen a subtraction algorithm, perhaps involving "borrowing"; we ask that you accept it as a valid method but that you do not promote it as the only method. After being introduced to "Nick's method", your students should begin to show a preference for an algorithm. Help students individually to gain facility with their preferences.

EN1 Minicomputer Introduction #1

CAPSULE LESSON SUMMARY

Introduce the values of the squares on the Minicomputer. Use the Minicomputer to represent numbers smaller than or equal to 100.

MATERIALS

Teacher: Minicomputer set*

Student: Pencil; Worksheets N1*, **, ***, and **** from UPG III

DESCRIPTION OF LESSON

Exercise 1

Display three Minicomputer boards.

T: Show me four fingers. Trace a "4" on your desk. That's one way to show 4. Here's another way.

Put one checker on the purple square of the ones' board.

T: This is called the "Minicomputer". With this checker, I put 4 on the Minicomputer.

* A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers.

EN 1

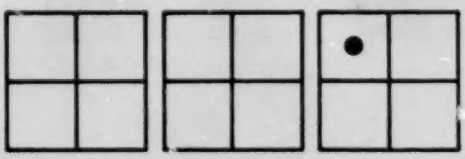
Move the checker from the purple square to the red square of the ones' board.

T: This is 2 on the Minicomputer.

Move the checker from the red square to the white square of the ones' board.

T: This is 1 on the Minicomputer.

Move the checker from the white square to the brown square of the ones' board.



T: Which number do you think this is? (Very often children will guess 3.) No, it's not 3. This is 8.

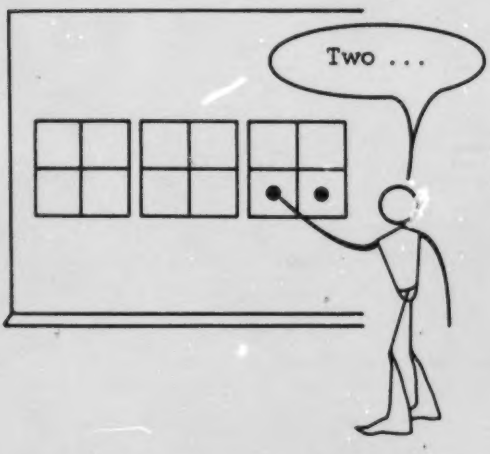
Review very briefly the placement of the checker for 1, 2, 4, and 8. Remove the checker from the Minicomputer.

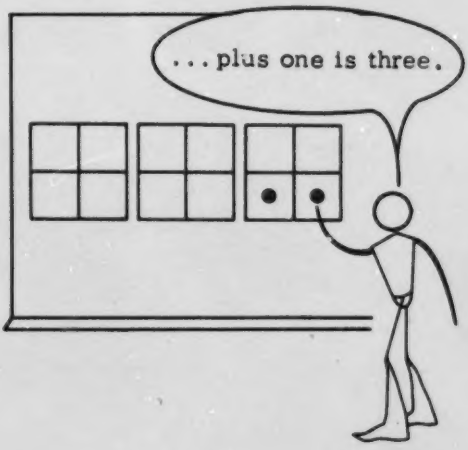
T: Who can put 3 on the Minicomputer?

Ask volunteers how many checkers they will need. (Two and three are both correct answers.) Let a student put 3 on the Minicomputer. If an answer is not correct, say so and ask for another volunteer. (See the illustration on the following page.)

After a student places the checkers this way, lift each checker as you mention its value and then replace it quickly on the Minicomputer.

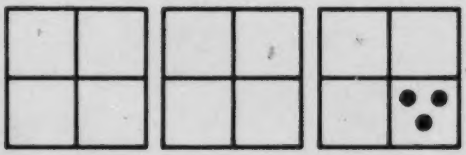
T: The red square is for which number? (2)
The white square is for which number? (1)
2 + 1 = 3. 1 + 2 = 3.





Remove the checkers from the Minicomputer.

NOTE: A student might put three checkers on the white square of the ones' board.



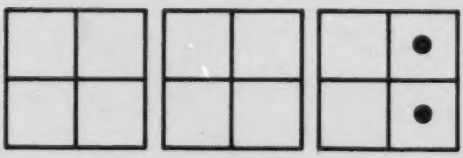
EN 1

If a student placed the checkers this way, lift the checkers one by one and say "1 + 1 + 1 = 3." Remove the checkers from the Minicomputer. Ask if someone can put 3 on the Minicomputer in another way.

T: Who can put 5 on the Minicomputer?

Ask a student who volunteers, how many checkers are needed. (Two, three, four, and five are all correct answers.) Ask the volunteer to place the checkers on the Minicomputer. As you did for 3, add the values of the checkers, each checker assuming the value of the square it is on. If the standard configuration is not suggested, continue asking for another way until you get the standard configuration.* You might find it helpful to ask for the same number using one checker fewer. For example, suppose a student named "Laura" put 5 on the Minicomputer using three checkers.

T: Laura put 5 on the Minicomputer using three checkers.
Can anyone put 5 on the Minicomputer using only two checkers?



When the checkers are in this position, lift each checker as you mention its value and replace it quickly on the Minicomputer.

T: The purple square is for which number? (4)
The white square . . . ? (1)
4 + 1 = 5. 1 + 4 = 5.

Remove the checkers from the Minicomputer.

---

* The standard configuration of a number on the Minicomputer has at most one checker on a square and has a digit 9 or less on each board. See Appendix A: STANDARD CONFIGURATION and STANDARD MINICOMPUTER MOVES at the end of the World of Numbers strand in UPG-III.

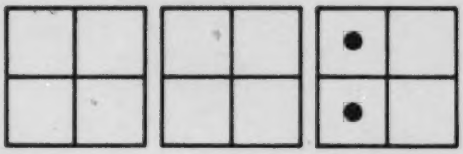
T: Can anyone put 6 on the Minicomputer?

T: Can anyone put 7 on the Minicomputer?

T: Can anyone put 9 on the Minicomputer?

Accept all correct answers. If the standard configuration is not suggested, continue asking for another way (the same number using fewer checkers) until you get the standard configuration. Remove the checkers from the Minicomputer.

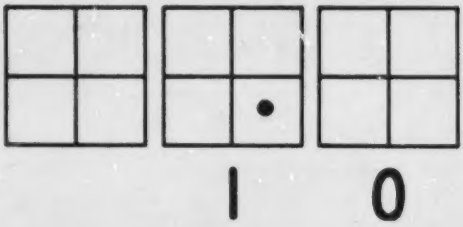
T: Can anyone put 10 on the Minicomputer?



T: There is a way to put 10 on the Minicomputer using only one checker.

EN 1

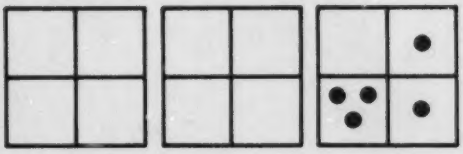
Place one checker on the white square of the tens' board. Write "1" below (or above if it is more convenient) the tens' board and "0" below the ones' board.



Erase "10" and remove the checker before going on to Exercise 2.

Exercise 2

Put several checkers on the ones' board; for example,



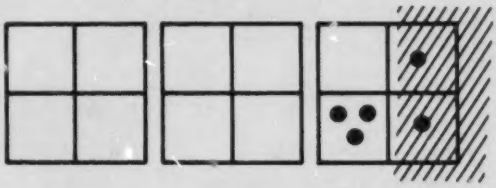
NOTE: Put the checkers gradually on the Minicomputer, for example,
first put three checkers on the red square, pause;
then put one checker on the purple square, pause again;
then put one checker on the white square.
This gives your students a chance to figure out mentally which number it is.

T: Which number is on the Minicomputer?

S: 11.

No explanation is necessary if everybody gets the right number. If someone gives the wrong number, you might use this procedure: cover part of the board with a piece of paper to focus the class' attention on certain checkers and gradually uncover the full set of checkers.

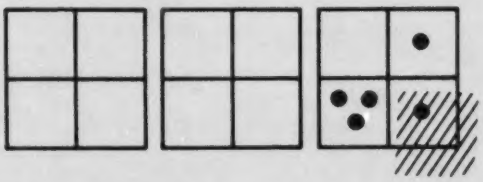
Cover the checkers on the purple and the white squares of the ones' board.



T: What number is this?

S: 6.

Uncover the checker on the purple square of the ones' board.



T: . . . and 4 more?

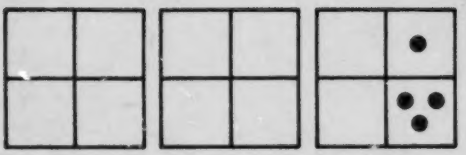
S: 10.

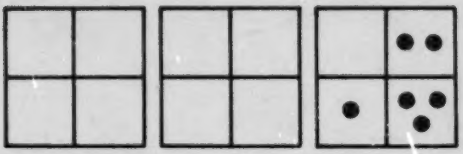
Uncover the ones' board completely.

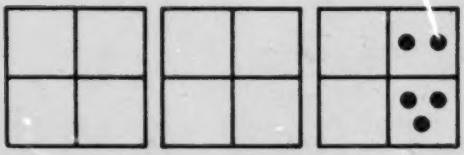
T: . . . and 1 more?

S: 11.

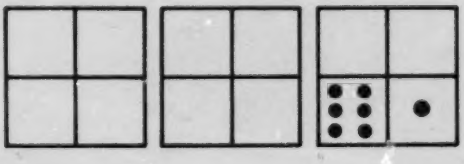
Repeat this activity with other examples; some possibilities are suggested on the next page.



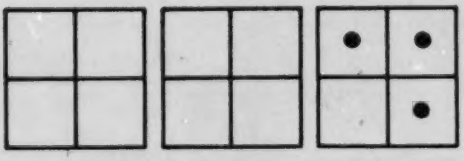




NOTE: It is interesting to present the same number several times, each time with a different number of checkers. For instance, 13 can be represented as:



or as



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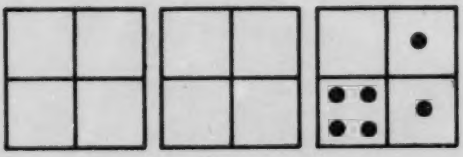
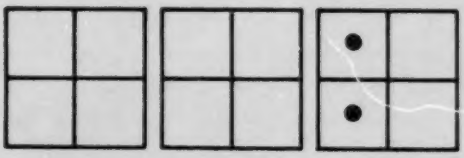


and so on.

Exercise 3

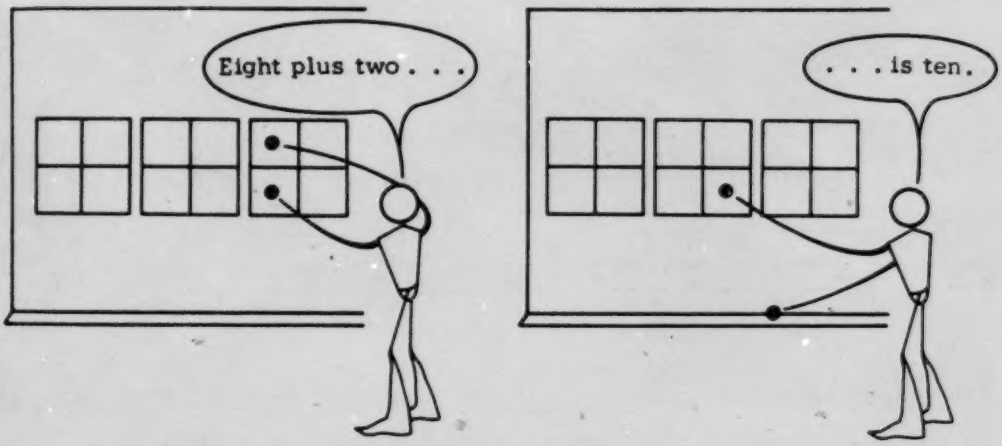
Put one checker on the 8-square and one on the 2-square.



T: What number is on the Minicomputer?

S: 10.

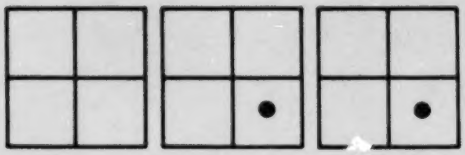
Pick up the checker on the brown square with one hand and the checker on the red square with the other. Then put one of these checkers on the tens' square and take the other checker away (put it in the chalk tray). As you are making the play, say, "8 + 2 = 10."*



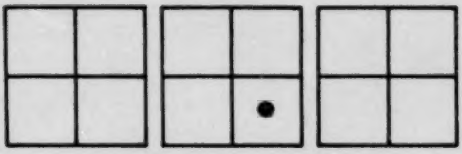
* The emphasis here is on the value of the squares on the tens' board, and not on plays.

EN 1

T: This is a way to put 10 on the Minicomputer using just one checker. I'll give you one more checker. Can you put 11 on the Minicomputer?



Remove the checker on the ones' board.



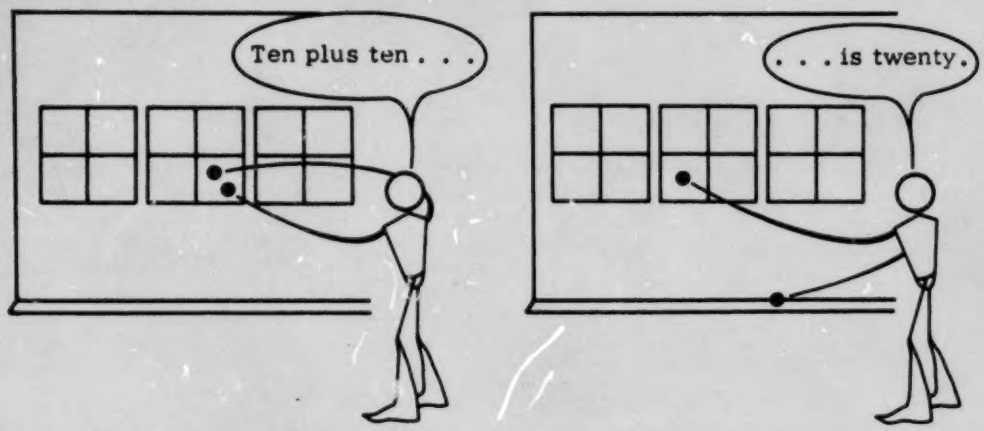
T: Using one more checker, can you show 12? (Yes)
Can you show 14? (Yes)
Can you show 17? (No)
How many checkers do you need? (At least three more)
Can you show 18? (Yes)
. . . 19? (No)

NOTE: The question "How many checkers do you need for 17?" has many answers. For the standard configuration you need four checkers, but it is possible to put 17 on the Minicomputer using only three checkers:
8 + 8 + 1 = 17.

Ask someone to put 20 on the Minicomputer using two checkers. If no one volunteers, put two checkers on the 10-square.

T: There is a way to put 20 on the Minicomputer using only one checker.

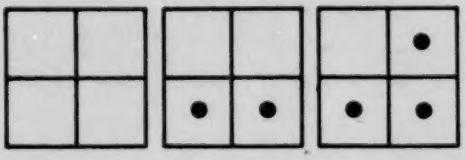
Make the play and say, "10 + 10 = 20."



Remove the checkers and continue with other numbers.

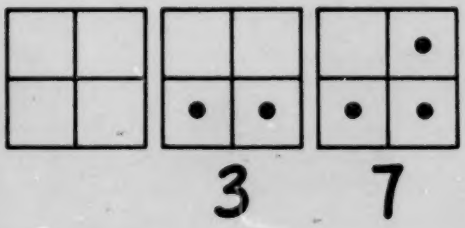
T: Who can put 25 on the Minicomputer? 27? 30?

Put this configuration on the Minicomputer.



T: What number is this?

When you receive the correct answer, write "3" below the tens' board and "7" below the ones' board.



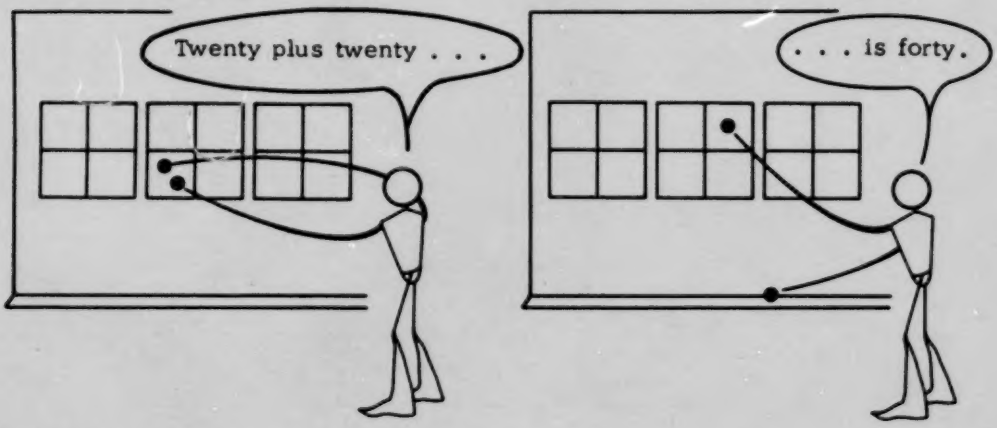
EN 1

Clear the Minicomputer and then put two checkers on the 20-square.

T: What number is this?

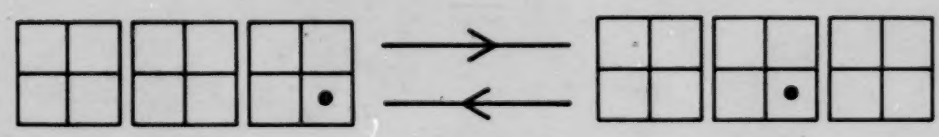
S: 40.

Make the play yourself and say, "20 + 20 = 40."



Remove the checker from the 40-square.

Move a checker back and forth very quickly from the 1-square to the 10-square. Each time you move the checker ask the class which number is on the Minicomputer: "one-ten; one-ten; and so on."



Repeat this with 2 and 20; 4 and 40; 8 and 80.*

T: Who can put 47 on the Minicomputer? 53? 96? 32?

Exercise 4

Give a student four checkers to place anywhere on the first two boards of the Minicomputer. You may want to stop a student who begins to put several checkers on the 8-square or the 80-square of the Minicomputer. Tell the class they can whisper to you what they think the number is. If appropriate, ask a student to write the number below the Minicomputer.

NOTE: Often in the description of a lesson it is suggested that you listen to whispers in order to give many students an opportunity to arrive at an answer independently. Let several students who have an answer whisper it in your ear. If appropriate, you will indicate to the student if the answer is correct. In some situations, for example detective stories, you will not indicate if the answer is correct because you want the student to continue to think about the situation.

If the number is too difficult for your students to read, read it yourself and continue the lesson.

Exercise 5

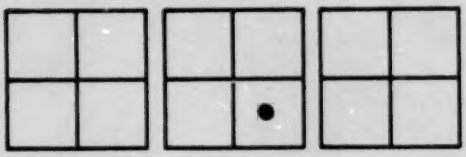
Put this configuration on the Minicomputer.

* Your students might need help with the introduction of the 80-square.

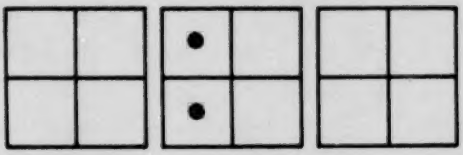
EN 1

T: What number is this? (10)

Who can put 10 on the Minicomputer using only one checker?



T: What number is this?

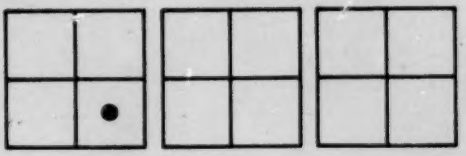


S: 100.

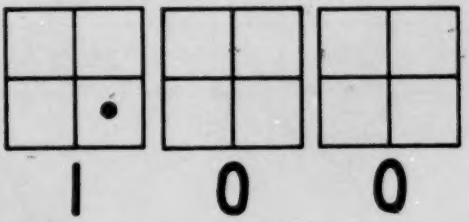
T: How do you know?

S: 80 + 20 = 100.

T: There is a way to put 100 on the Minicomputer using just one checker.



Make the play yourself and say, "80 + 20 = 100." Write "100" below the Minicomputer.



T: Who can put 105 on the Minicomputer? 126?

Occasionally, you may wish to have a student write a numeral below the Minicomputer.

Your students should do all or some of the Worksheets N1*, **, ***, and **** in the Worksheet booklet for UPG III. N1 in UPG III is similar to EN1, and the worksheets for N1 are appropriate for either lesson. The answer key for these worksheets can be found on page 13 in the World of Numbers strand of UPG III.

EN2 The Functions +5 and -5

CAPSULE LESSON SUMMARY

Practice counting by fives. Collectively, complete an arrow picture with +5 arrows and another with -5 arrows making use of return arrows. Individually, draw other arrow pictures using +5 and -5 arrows.

MATERIALS

Teacher: Colored chalk
Student: Colored pencils*; pencil; unlined paper

DESCRIPTION OF LESSON

Start the lesson with a mental arithmetic exercise of counting by fives from 5 to 50.

Draw this arrow picture on the board.

T: In this arrow picture the dots are for numbers and the arrows are for +5. Where is the smallest number in this picture?

Let someone come to the board and point to the dot for the smallest number. (The dot furthest to the left.)

T: What is the smallest number in this picture?

* Crayons can be substituted for colored pencils in any of the lessons.

S: 11.

T: Who can come to the board and point to the dot for the largest number? (The dot furthest to the right) What is the largest number in this picture? Tell me with whispers. (31)

Take whispers from several students and then ask someone to give the answer aloud.

Point to the dot to the right of 16.

T: What number is here? How do you know?

S: 21, because 16 + 5 = 21.

Label the dot "21" and then point to the next dot to the right.

T: What is this number? How do you know?

S: 26, because 21 + 5 = 26.

Label this dot and ask for a volunteer to label the last dot to the right.

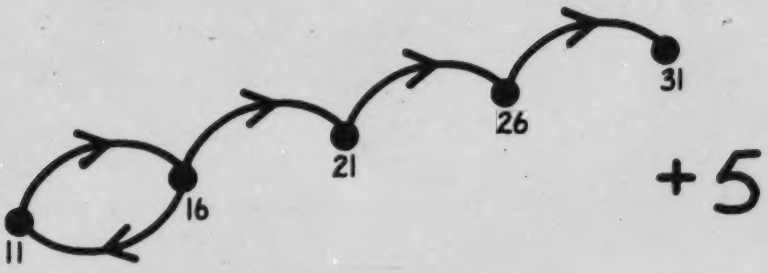
T (pointing to the dot furthest to the left): What number is here? How do you know?

S: 11, because 16 - 5 = 11 (or 11 + 5 = 16).

If someone answers "11 + 5 = 16", accept it but ask for another explanation.

EN 2

Draw a return arrow in blue from 16 to 11.



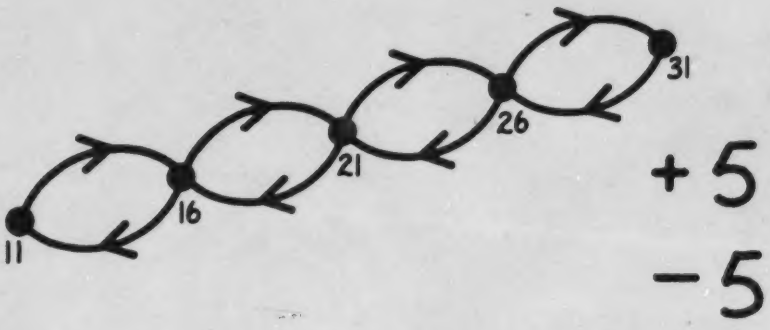
T: What could the blue arrow be for?

S: -5.

Write "-5" in blue near the arrow picture. As you trace the arrow from 16 to 11, say:

T: 16 - 5 = 11.
Could we draw any more blue arrows in this picture?

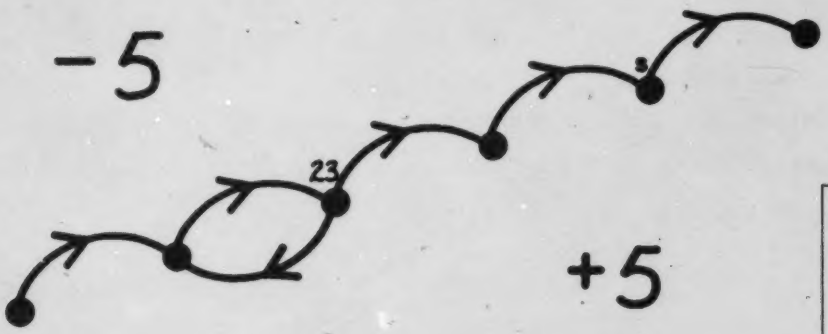
Complete the picture quickly with your class.



T: If we continue drawing red arrows (to the right), would we ever meet 51? (Yes) 63? (No)
How do you know?

Accept any response which is reasonably correct. Encourage looking for patterns.

Erase the board and draw this arrow picture.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: What are the blue arrows for? (-5)
Where is the smallest number in this arrow picture?
Where is the largest number in this arrow picture?

Point to the dot labeled "s" in the illustration above.

T: Tell me with whispers what number is here. (13)

Take whispers from several students and then ask someone to answer aloud.

T: Who can label a dot?

Occasionally, ask students to explain how they know which numbers correspond to particular dots. Encourage students to use return arrows.

T: If we go on drawing more red arrows (to the left), do you think we would ever meet 50? (No) How do you know?

Encourage students to observe a pattern — the numbers which are met all end in 3 or 8. Continue these questions if the class is interested.

Erase the board and distribute unlined paper. Ask the students to draw their own arrow pictures using +5 arrows. Allow them to begin with a number of their own choosing and encourage them to continue as long as possible. Take time to help the students set up their pictures. Stress the importance of a "key" in a picture to tell what the arrows are for. Instruct students to draw dots and numerals in pencil, arrows and keys in color. Also mention that for any one number in a picture they need to draw only one dot. Students who quickly finish a rather extensive picture could be asked to find and draw -5 arrows between numbers already in their pictures.

EN3 Minicomputer Introduction #2

CAPSULE LESSON SUMMARY

Find many ways to display the same number on the Minicomputer.

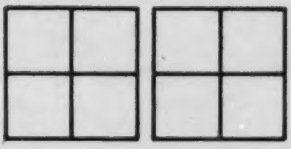
MATERIALS

Teacher: Minicomputer set

Student: Minicomputer set*

DESCRIPTION OF LESSON

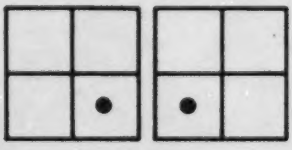
Display two Minicomputer boards and review the values of the squares.

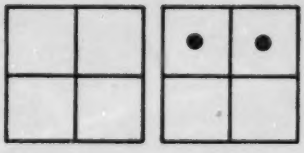


Distribute two individual Minicomputer boards and some checkers to each student. Display two Minicomputer boards. Ask the students to put 15 on their (desk) Minicomputers. When most of the students have finished, ask someone to put 15 on the demonstration Minicomputer.

Repeat this activity with the number 51.

Ask the students to put 12 on their Minicomputers using exactly two checkers. Let students put 12 on the demonstration Minicomputer and ask them to explain their solutions. There are two ways to show 12 using exactly two regular checkers.



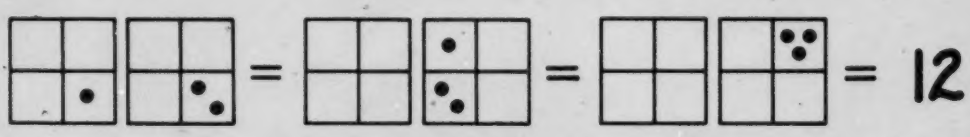


* A student's Minicomputer set consists of two individual Minicomputer sheets (each having two boards) and a sheet of individual checkers.

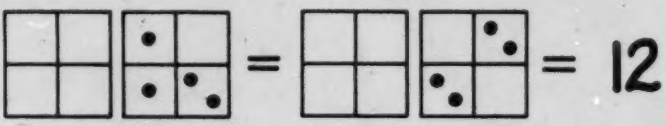
EN 3

Continue this activity showing 12 on the Minicomputer with three checkers, with four checkers, and with five checkers. Solutions for each of these cases are illustrated below. Do not expect your class to find all of the solutions in each case.

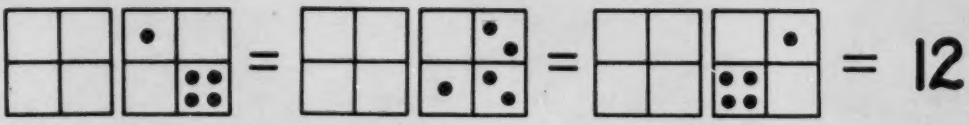
Three checkers:



Four checkers:

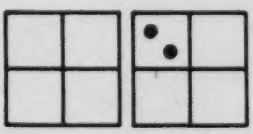


Five checkers:

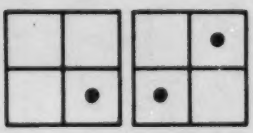


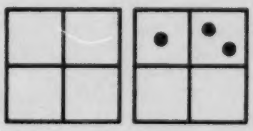
Repeat this activity with the number 16. Solutions are shown below.

Two checkers:

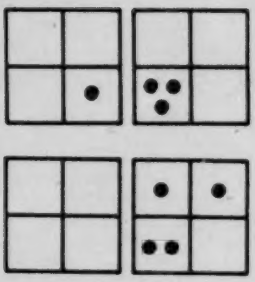


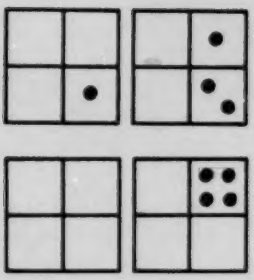
Three checkers:



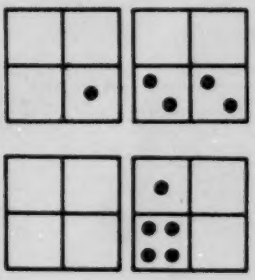


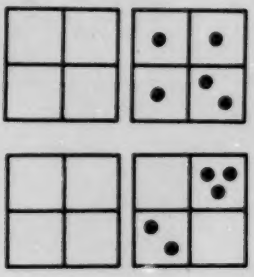
Four checkers :





Five checkers :





EN 4

EN4 Roads

CAPSULE LESSON SUMMARY

Build a road from one number to another using two types of arrows.

MATERIALS

Teacher: Colored chalk

Student: Colored pencils; pencil; unlined paper

DESCRIPTION OF LESSON

Draw two dots well spaced on the board. Label one of the dots "6" and the other "11".

T: Today we are going to build roads. These roads are made up of arrows. Let's begin by building a road from 6 to 11. The blue arrows are for +2 and the red arrows are for -1.

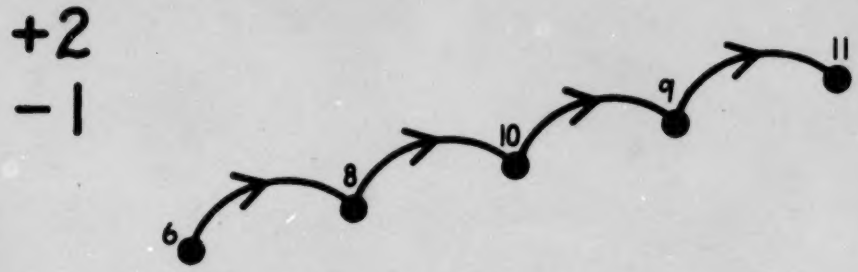
You might refer to "+2" and "-1" as the "key" or "color code" for this road.

T: Which kind of arrow would you like to start with?

Take whatever suggestion is made (e.g. blue) and draw the first arrow yourself. Point to the ending dot of this arrow.

T: Which number is here? (8, if the arrow is blue; 5, if the arrow is red)

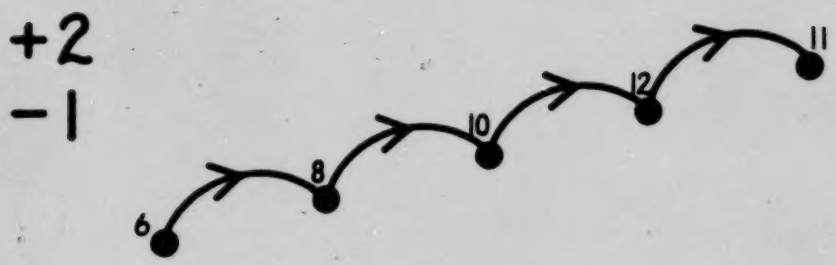
Ask students to complete the road one arrow at a time. One of the many possible roads is shown below.



NOTE: If some students are concentrating on a color pattern for the arrows, you might encourage them to keep in mind what the target number is. In this particular example, alternating +2 and −1 arrows <u>will</u> build a road from 6 to 11 using ten arrows, five blue and five red. There is nothing wrong with such a road, but you might ask if it is possible to build a road using fewer arrows. In this case, there is a shorter road as illustrated above.

EN 4

There is no rule against "overshooting" the larger number. For example, here is a perfectly acceptable road.



Construct two or three different roads from 6 to 11 with your class. The purpose of this first collective exercise is to show what a road is and that many different solutions are possible.

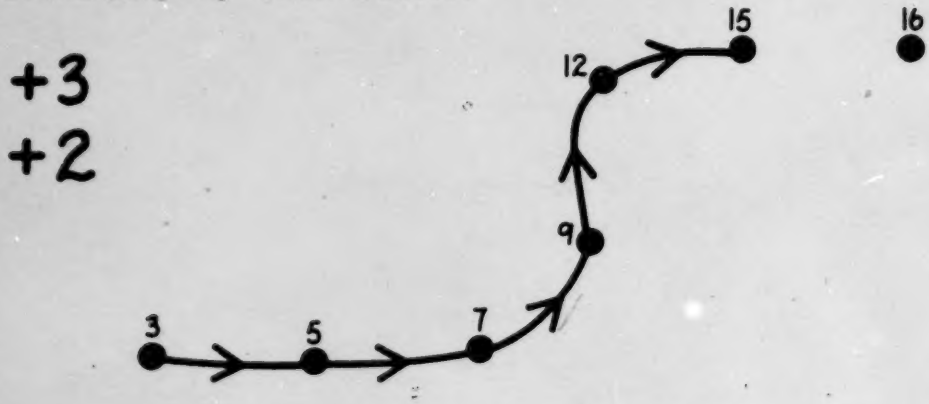
Distribute paper and colored pencils and put the following information on the board.

Ask the students to copy what is on the board and to build a road from 6 to 21 using +2 and +1 arrows. Walk around the room as the students work, giving assistance to those who need it. Students who draw cramped pictures should be encouraged to use the whole sheet of paper. As the rest of the class continues to work, ask a couple of students with different numbers of arrows in their roads to copy their pictures on the board. When most of the students have finished, direct the class' attention to the examples on the board. Check each picture on the board with the class to see that it is correct. Point out that there are many correct solutions to this problem.

Afterward, ask which road on the board has the fewest arrows and determine if anyone made a road with even fewer arrows.

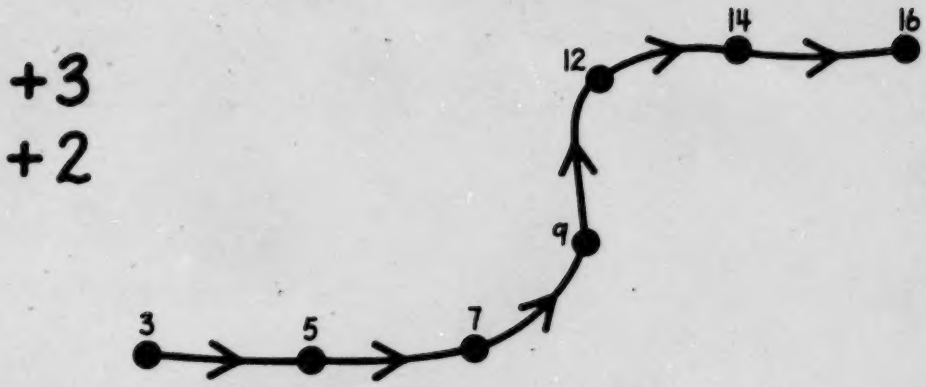
Repeat this activity building a road from 3 to 16 using +3 and +2 arrows.

NOTE: A student might draw this road and discover that one can not reach 16 from 15 using only +3 and +2 arrows.



In such a case, you should suggest changing the color of the last arrow. In the preceding picture, changing the arrow starting at 12 from red to blue would enable the student to complete a road with one additional blue arrow.

EN 4



If there is time remaining, ask the students to build one or more of the following roads.

1) Build a red-blue road from 17 to 30 using **+1** and **+2** arrows.

2) Build a red-blue road from 8 to 19 using **+3** and **−2** arrows.

3) Build a red-blue road from 10 back to 10 using **+3** and **−2** arrows.

4) Build a red-blue road from 27 back to 27 using **+3** and **−2** arrows.

EN 5 Minicomputer Introduction #3

CAPSULE LESSON SUMMARY

Practice reading numbers on the Minicomputer. Identify some Minicomputer plays and use them in addition, subtraction, and multiplication calculations.

MATERIALS

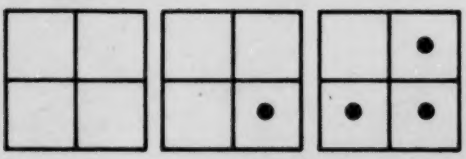
Teacher: Minicomputer set
Student: None

DESCRIPTION OF LESSON

Exercise 1

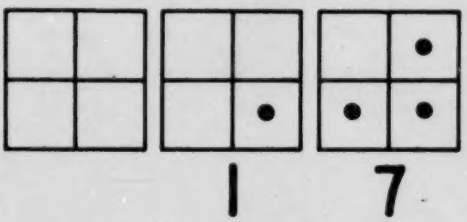
This activity should move quickly. Choose numbers which are appropriate for your class and encourage slower students to participate.

Display three Minicomputer boards.

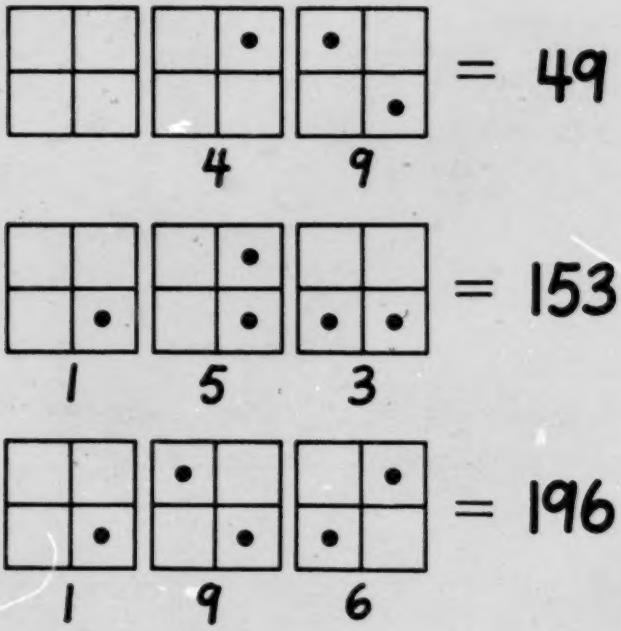


T: What number is on the Minicomputer? (17)

Who can write "17" below the Minicomputer?



Repeat this activity with other numbers such as those illustrated below. Each time, ask the students to identify the number and then ask someone to write the numeral below the Minicomputer.



Change the activity so that students put numbers on the Minicomputer.

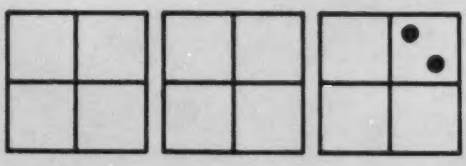
T: Who can put 46 on the Minicomputer? 57? 124?

Move checkers quickly from one board to another to review the standard configurations for these numbers.

1, 10, 100	8, 80, 800
2, 20, 200	5, 50, 500
4, 40, 400	3, 30, 300

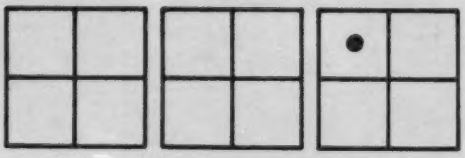
Exercise 2

Put two checkers on the 4-square of the Minicomputer.

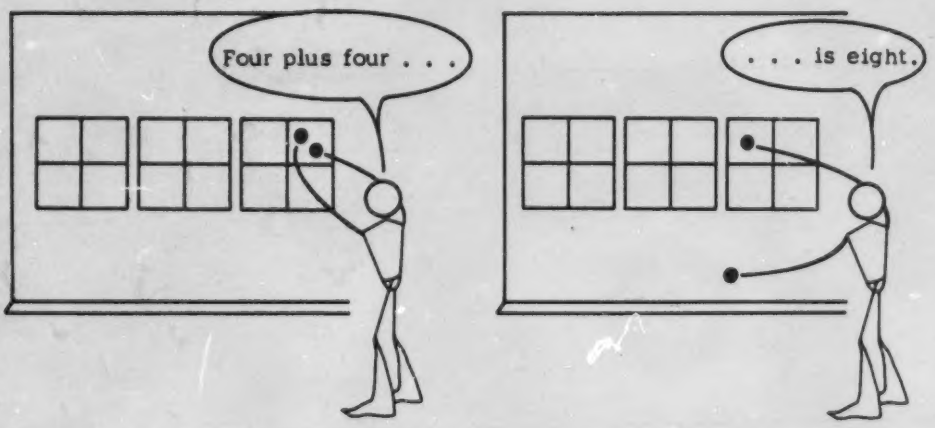


T: What number is this? (8)

Can someone put 8 on the Minicomputer using just one checker?



Repeat the play emphatically and say, "4 + 4 = 8."



Then make the backward play "8 = 4 + 4".

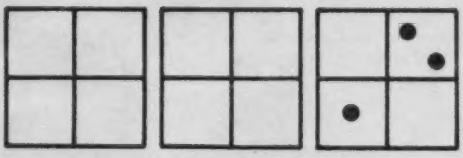
EN 5

Similarly, demonstrate the following plays:

40	+	40	=	80	and	80	=	40 + 40
100	+	100	=	200	and	200	=	100 + 100
200	+	200	=	400	and	400	=	200 + 200
8	+	2	=	10	and	10	=	8 + 2

Exercise 3

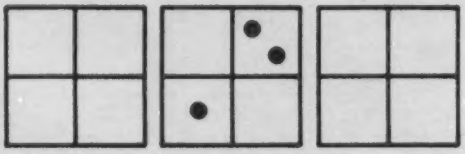
Put this configuration on the Minicomputer.



T: What number is this? (10)

Invite students to make plays until the standard configuration for 10 is on the Minicomputer. Students should name plays before moving any checkers.

Repeat the exercise with this configuration.



T: What number is this? (100)
What do you notice?

Encourage the students to comment on this new situation and to explain why this number is 100 before asking for volunteers to make plays.

Exercise 4

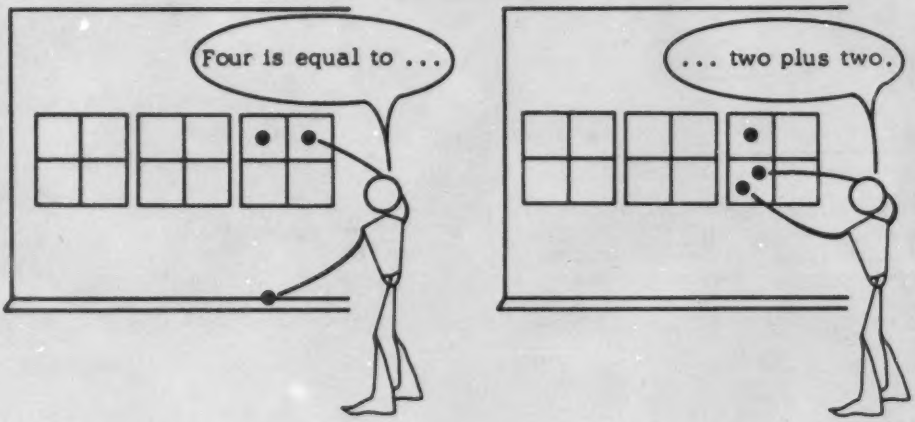
Put this configuration on the Minicomputer.

T: What number is this?

S: 12.

T: Without changing the number, I would like to get a checker on the red square so that we can make the jump to the tens' board. What play should we make?*

If the "4 = 2 + 2" play is not suggested, make the play yourself.



* It is possible that a student will suggest an "8 + 4 = 10 + 2" play. Although such plays are not emphasized, they should not be discouraged. In this case, you are interested in demonstrating the "4 = 2 + 2" play. If another valid play is suggested, you should indicate that it is correct, but that you would prefer the "4 = 2 + 2" play for this activity.

T: I want someone to make a play so we will have a checker on the tens' board.

Ask the students to name the play they intend to make before they move any checkers. Do not accept the "2 + 2 = 4" play because you have asked for a play which results in a checker on the tens' board.

NOTE: Backward plays may be difficult for your students at this time. There will be many opportunities to work on these moves in the future.

Exercise 5: Addition Problems

Write this problem on the board.

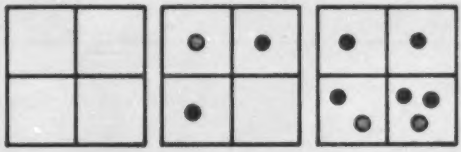
$$47 + 29 + 83$$

Lead the class to estimate this sum by asking for numbers which the sum is larger than and for numbers which the sum is smaller than. Perhaps your class will estimate it to be between 140 and 160.

T: How do you think we add numbers on the Minicomputer?

S: First, put on each number.

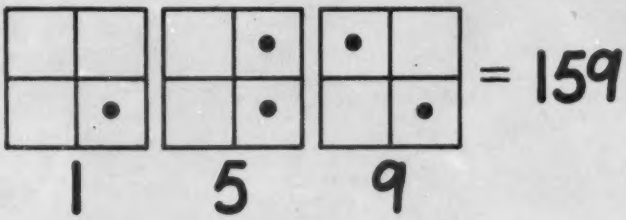
Invite three students to put the numbers 47, 29, and 83 on the Minicomputer. To emphasize which three numbers are being added, ask that each number be displayed in a different color, although usually no significance is attached to the color of the checkers.



T: What could we do to make the answer easier to read?

S: Make plays.

If the answer is not forthcoming, suggest it yourself. Invite students one at a time to make plays until the standard configuration for 159 is obtained. Students should name plays before moving any checkers.



Conclude that 47 + 29 + 83 = 159.

Exercise 6: Subtraction Problems

Write this problem on the board.

653 - 412

T: Do you think 653 - 412 is more or less than 653?

S: Less, because subtracting 412 from 653 gives a smaller number.

T: Is it less than 500?

S: Yes, 600 - 400 = 200.

T: Is 653 - 412 more or less than 200?

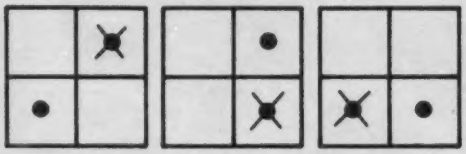
S: More, 600 - 400 = 200 and we still have 53 and are only taking away 12.

EN 5

T: Who can put 653 on the Minicomputer?
Who can take away 412?

NOTE: When students are asked to put numbers on the Minicomputer, they may or may not show standard configurations for them. The lesson descriptions assume that standard configurations of numbers will be put on the Minicomputer. Adjust the lessons depending upon which configurations your students display.

A student should remove the checker from the 400-square, the checker from the 10-square, and the checker from the 2-square.



Ask a student to write the answer below the Minicomputer and conclude that 653 - 412 = 241.

Exercise 7: Multiplication Problems

Write this problem on the board.

$$2 \times 37$$

Read the problem and review (or introduce) that "2 x 37" is the same number as "37 + 37". Let the students guess the answer.

T: Who would like to put 37 on the Minicomputer using blue checkers?
Who would like to put 37 on the Minicomputer again only this time

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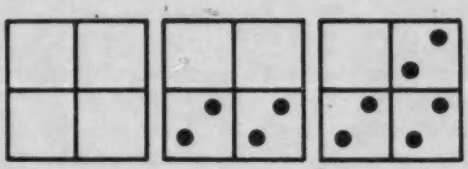
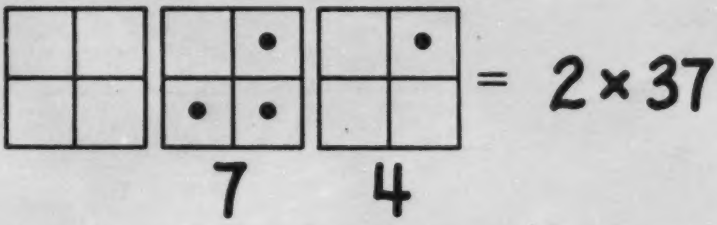
* An "X" through a checker indicates that the checker has been removed from the Minicomputer.

using red checkers?



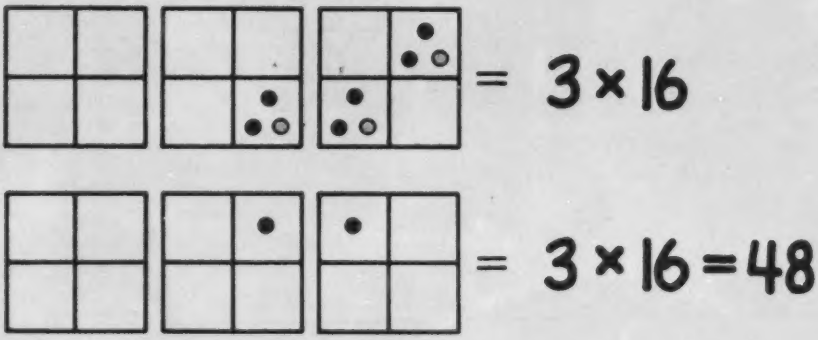
T: Who can make some plays so the answer will be easier to read?

Invite students to make plays until the standard configuration for 74 is obtained. Students should name plays before moving any checkers.



Conclude that 2 x 37 = 74.

Repeat this activity to compute "3 x 16" on the Minicomputer.



EN 6

EN6 Minicomputer Introduction #4

CAPSULE LESSON SUMMARY

Represent a sum on the Minicomputer by the use of checkers of two different colors. Explore the effect of adding a board to the right. Present a situation where a backward play is used to arrive at the standard configuration of a number. Explore the effect of moving various checkers in a configuration on the Minicomputer -- in each case, does the move increase, decrease, or leave the same number on the Minicomputer? Review plays on the Minicomputer as a way to make numbers easier to read.

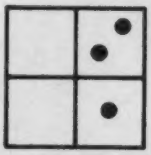
MATERIALS

Teacher: Minicomputer set
Student: None

DESCRIPTION OF LESSON

Exercise 1

Display one Minicomputer board.



(Use two different colors for the checkers.)

T: What number is on the Minicomputer?

S: 9.

T: How do you see 9?

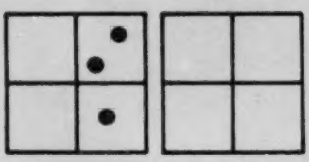
Let students make suggestions, and if it is not suggested that 9 can be seen as 4 (in red) +5 (in blue), ask the following question.

T: What number is on the Minicomputer with red checkers? (4)

. . . with blue checkers? (5)

Write "4 + 5 = 9" on the board.

Without removing the checkers, add a second Minicomputer board to the right.



T: Now, what number is on the Minicomputer?

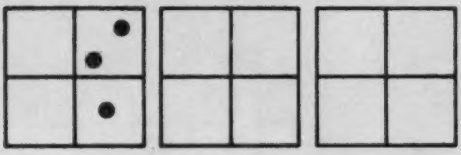
S: 90.

T: How do you see 90?

S: 40 in red plus 50 in blue.

Write "40 + 50 = 90" on the board.

Add a third board to the right.



T: What number is on the Minicomputer?

S: 900.

T: How do you see 900?

S: 400 in red plus 500 in blue.

EN 6

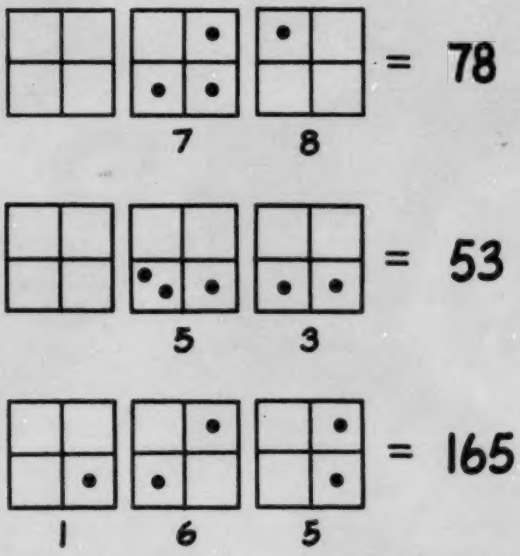
Write "400 + 500 = 900" on the board.

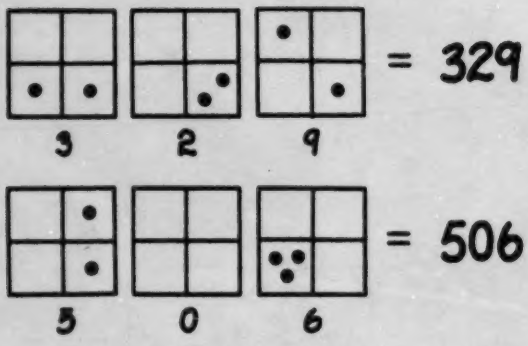
If your class seems to enjoy this exercise, you may go on adding boards (or drawing more boards on the chalkboard when you run out), and asking students to write the calculations:

$$
\begin{aligned}
4 + 5 &= 9 \\
40 + 50 &= 90 \\
400 + 500 &= 900 \\
4{,}000 + 5{,}000 &= 9{,}000 \\
40{,}000 + 50{,}000 &= 90{,}000.
\end{aligned}
$$

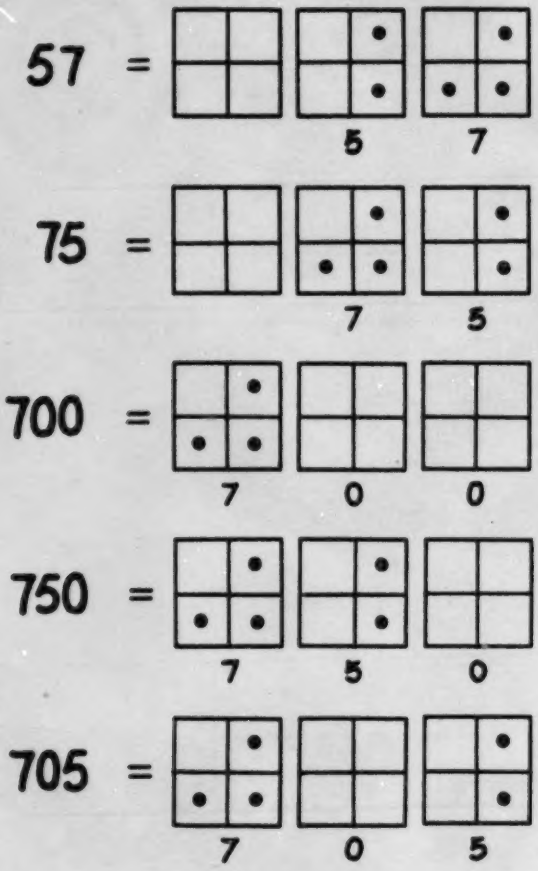
Exercise 2

Display three Minicomputer boards. Ask the students to read various numbers you put on the boards and to write the numerals with each digit centered below the appropriate board.





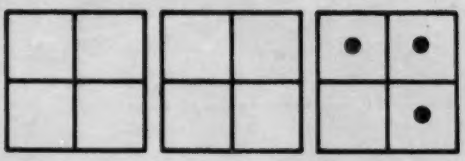
Vary the activity by asking the students to put various numbers on the Mini-computer and again to write the numerals below the boards. A good sequence of numbers you could ask for is suggested here.



EN 6

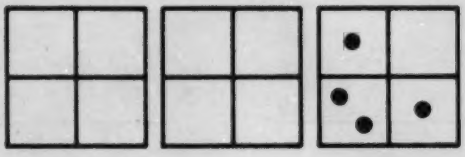
Exercise 3

Put this configuration on the Minicomputer.



T: What number is this? (13)
Without changing the number, I want to get a checker on the red square so I can make the jump to the tens' board. What play should I make?

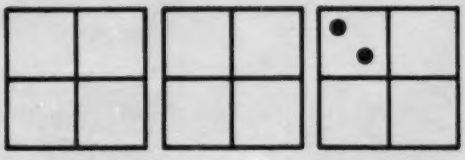
If no one suggests "4 = 2 + 2", suggest it yourself and make the play.



T: Who can make a play so we will have a checker on the tens' board?

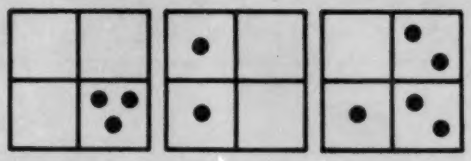
Do not accept the "2 + 2 = 4" play since you have ask for a play which results in a checker on the tens' board.

Repeat this exercise with this configuration.



Exercise 4: Transforming a Number

Put this configuration on the Minicomputer.



T: I put a number on the Minicomputer.
Is this number larger than 100? How can you tell?

S: Yes, because there is 300 on the hundred's board.

T: Is this number larger than 300? How can you tell?

S: Yes, because there is 300 on the hundreds' board and more checkers on other boards.

T: Is this number larger than 400? How can you tell?

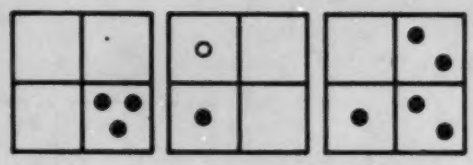
S: Yes, because there is 100 on the middle (tens') board and 300 + 100 = 400 and there are still more checkers on the ones' board.

(To answer these questions, your students could point to some of the checkers or could suggest some moves.)

T: For the moment, do not try to find out exactly what number it is. I will move, remove, or add some checkers. Each time, tell me if I get a larger number, a smaller number, or the same number.

EN 6

Replace one of the checkers by a checker of another color.



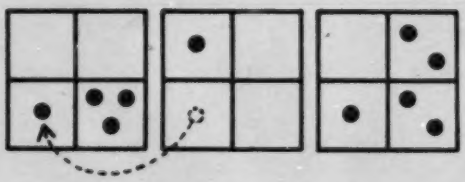
T: What do you think?

S: The same.

T: Why?

S: Because all you did was exchange a checker for a different color checker.

Move the checker from the 20-square to the 200-square.



T: Larger, smaller, or the same?

S: Larger.

T: Why?

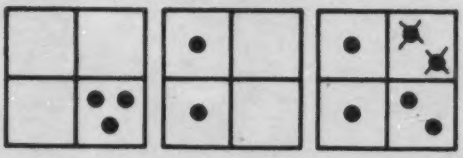
S: Because you moved the checker on the 20-square to the 200-square.

Return the checkers to their original positions.

T: Who can move one checker to get a smaller number?

Ask a student to do this and discuss why the move makes the number smaller. Then return the checkers to their original positions.

Remove two checkers from the 4-square and put one checker on the 8-square.



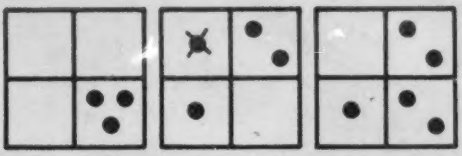
S: The same.

T: Why?

S: 4 + 4 = 8.

Return the checkers to their original positions.

Remove the checker from the 80-square and put two checkers on the 40-square.



S: The same.

T: Why?

S: 80 = 40 + 40.

Return the checkers to their original position.

T: How can we make the number easier to read?

S: Make some plays.

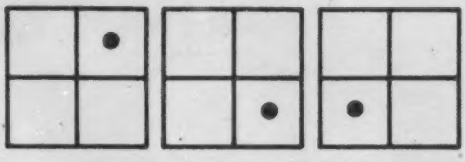
Ask students to name the play they intend to make before they move any checkers. For example,

S: 4 + 4 = 8.

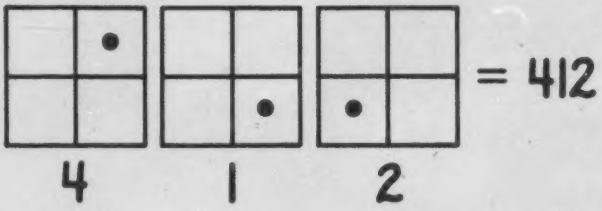
S: 100 + 100 = 200.

Be sure the students understand that any play they make must leave the number the same. Be prepared to make a backward play yourself if one is needed and the students do not suggest one.

Continue until the standard configuration for 412 is obtained.

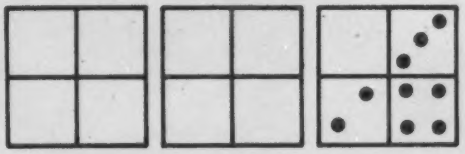


Read the number with the students and write it below the Minicomputer.



Exercise 5

Put the checkers on the Minicomputer gradually, allowing the students to calculate the number mentally.



T: What number is on the Minicomputer?

S: 20.

T: How many checkers have I used?

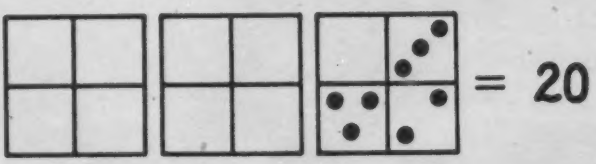
S: Nine.

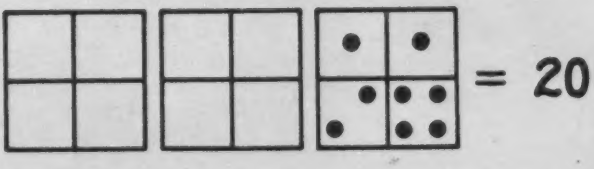
T: What number is 1 less than 9?

S: 8.

T: Can you put 20 on the Minicomputer using one checker fewer (i.e. using eight checkers)?

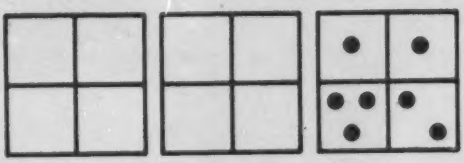
If the students do not respond, suggest that they make a play. Any forward play they make will solve the problem. For example,





Other solutions are also possible.

T: This is 20 with eight checkers.
Can you put 20 on the Minicomputer using exactly seven checkers?



Other solutions are also possible.

EN 6

T: Can you put 20 on the Minicomputer using one checker fewer (i.e. using six checkers)?

Do not overextend this activity, but return to it for a few minutes from time to time during the first months of the year.

EN7 The Functions +3 and -3

CAPSULE LESSON SUMMARY

Count forward and backward by threes. Examine the relationship between the number stories "1 + 3 = 4" and "4 - 3 = 1" by the use of a simple arrow diagram. Make use of return arrows to label some of the dots in an arrow picture using +3 arrows and in another using -3 arrows. Individually, draw arrow pictures using +3 and -3 arrows.

MATERIALS

Teacher: Colored chalk
Student: Colored pencils; pencil; unlined paper

DESCRIPTION OF LESSON

Exercise 1

Divide your class into three or four groups in a manner natural to the seating arrangement. Indicate an order in each group, i.e., which student is first, second, and so on.

T: We are going to count forward by threes starting at 0. I will start by saying "0", then the first student in a group will say "3"; the next, "6"; the next,"9" because 9 is three more than 6; and so on. Try to remember the number you say.

Let each group take its turn.

Now reverse the counting. Starting with the last student in each group, count backward by threes by asking students to recall their numbers. For example, a group of seven students would count:

21, 18, 15, 12, 9, 6, 3

. . . and you would say "0".

If a student is having difficulty with either activity, hold up three fingers and count (by ones) forward or backward, as the case may be, from the previous number.

If your class is capable, you may wish to have the entire class participate as a single group in counting forward and backward by threes.

Exercise 2

Draw this arrow picture on the board.

T: What is this red arrow for?

S: +3.

T (pointing to the dots): Do you know what these numbers are?

S: No.

T (pointing to the dot on the left): If this number were 5 (trace the red arrow and point to the dot on the right), what number would this be?

S: 8, because 5 + 3 = 8.

T (pointing to the dot on the left): If this number were 8, what would the other be? (11)
If this number were 0 . . . ? (3)
If this number were 14 . . . ? (17)
If this number were 100 . . . ? (103)

Very obviously, indicate a change in the questioning.

T (pointing to the dot on the right): If this number were 4, what would the other number be? How do you know?

S: 1, because 4 - 3 = 1.

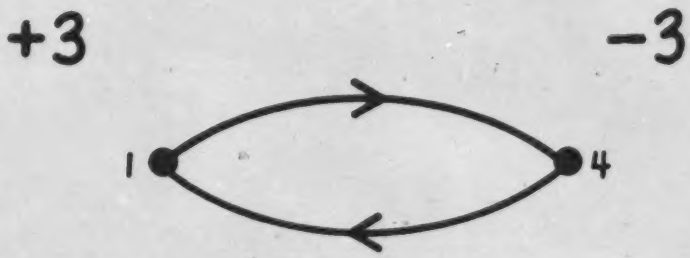
If the student answers "1 + 3 = 4", ask if someone could find the number another way.

Label the dots and trace an arrow from 4 to 1.

T: If we draw a blue arrow from 4 to 1, what could it be for?

S: -3.

Draw the return arrow in blue and write "-3" in blue near the arrow picture.



T: What number stories are told by this arrow picture?

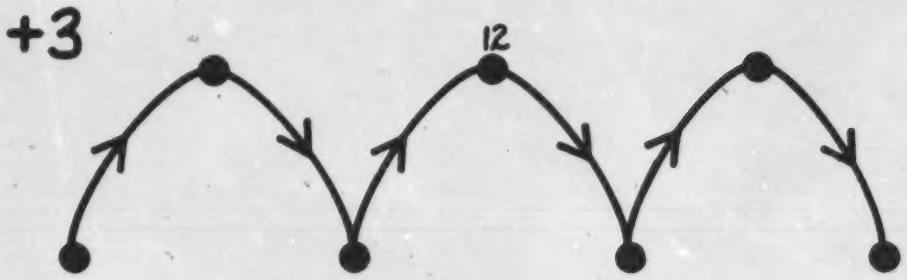
S: 1 + 3 = 4 and 4 - 3 = 1.

Erase "1" and "4". Alternating dots, assign a number to a dot and ask what the other number in the picture would be. Adjust the difficulty of the calculations to the abilities of your students.

EN 7

Exercise 3

Draw this arrow picture on the board.



T: Point to the largest number in this arrow picture. Tell me with whispers what the largest number in this arrow picture is.

S: 21.

T: Point to the smallest number in this arrow picture. Tell me with whispers what the smallest number in this arrow picture is.

S: 3.

It is not necessary for you to reveal immediately the identities of the smallest and largest numbers to the class. When these dots are labeled, acknowledge any students who had whispered the correct numbers to you earlier and ask how they found their answers.

Ask for volunteers to label the dots in this arrow picture. Occasionally ask students to explain how they know which numbers correspond to particular dots. Continue until the dot to the left of 12 is labeled "9". Trace the red arrow from 9 to 12.

T: How do you know that the starting number for this arrow is 9?

S: 12 - 3 = 9.

If a student answers "9 + 3 = 12," ask if someone can find this number another way.

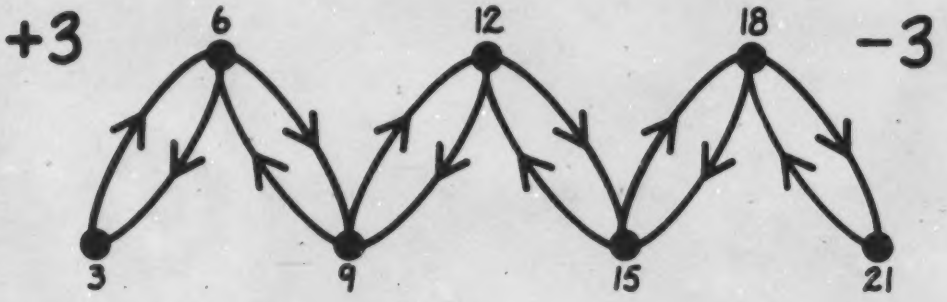
Draw a blue arrow from 12 to 9.

T: What could this blue arrow be for?

S: -3.

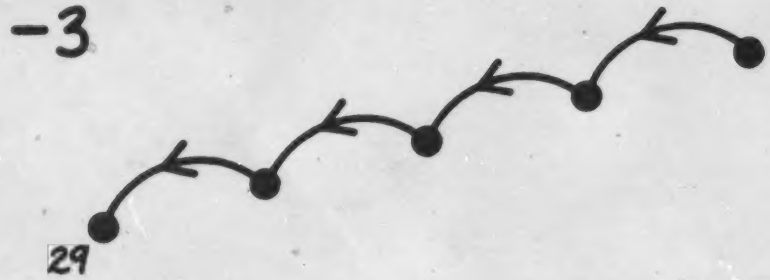
Write "-3" in blue near the arrow picture.

Invite students to draw return arrows and label dots until the arrow picture is complete.



Exercise 4

Draw this arrow picture on the board.



EN 7

T: Point to the smallest number in this arrow picture. What is the smallest number in this picture?

S: 29.

T: Point to the largest number in this arrow picture. Tell me with whispers what the largest number in this picture is.

S: 41.

Ask students to explain their methods of finding the largest number.

T: This arrow road ends at the number 29. What can we do that will help us calculate these other numbers?

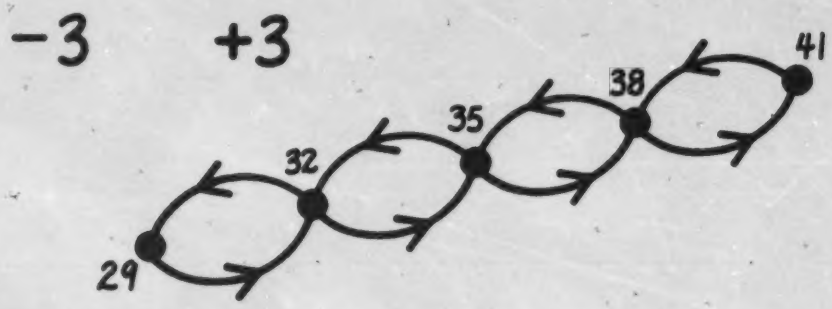
S: Draw return arrows.

You may need to suggest drawing return arrows yourself.

T: What could the return arrows be for?

S: +3.

Write "+3" in red near the arrow picture and invite students either to draw a +3 arrow or to label a dot. Complete the arrow picture.



Distribute red pencils, blue pencils, and pieces of unlined paper. Ask the students to draw their own arrow pictures using +3 arrows and when they are finished to draw -3 arrows between the numbers already in their pictures. Ask students who finish an extensive picture quickly to draw another arrow picture labeling the dots with larger numbers.

EN 8

EN8 The Functions $2x$ and $1/2x$

CAPSULE LESSON SUMMARY

Relate the functions $2x$ and $\frac{1}{2}x$ through the use of a simple arrow diagram: the arrow for one is the return arrow for the other. Label the dots in a $2x$ arrow picture with the help of the Minicomputer, and then draw the return arrows.

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: None

DESCRIPTION OF LESSON

Exercise 1

Begin the lesson with some mental arithmetic.

T: What number is 10 + 10? (20)
Do you know another name for 10 + 10? ("2 x 10")
What number is 2 x 10? (20)
What number is $\frac{1}{2}$ x 20? (10)

What number is 50 + 50? (100)
What number is 2 x 50? (100)
What number is $\frac{1}{2}$ x 100? (50)

What number is 300 + 300? (600)
What number is 2 x 300? (600)
What number is $\frac{1}{2}$ x 600? (300)

NOTE: "$\frac{1}{2}$ x" should be read "one half of"; for example, "$\frac{1}{2}$ x 20" is read "one half of twenty."

Continue the exercise with these calculations.

$2 \times 30 \quad (60)$

$\frac{1}{2} \times 60 \quad (30)$

$2 \times 25 \quad (50)$

$\frac{1}{2} \times 30 \quad (15)$

$2 \times 16 \quad (32)$

$\frac{1}{2} \times 32 \quad (16)$

Draw this arrow picture on the board.

Point to the dot on the right.

T: What number is this?

S: 30.

Label the dot on the right "30" and draw the return arrow from 30 to 15 in blue.

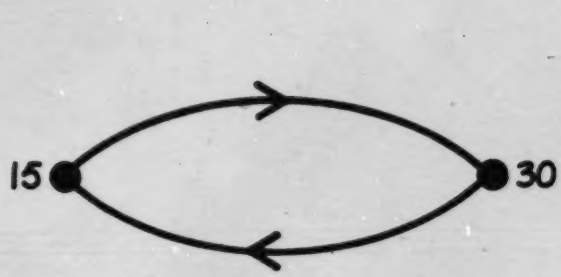
T: What could this blue arrow be for?

S: $\frac{1}{2}$ x.

If a student suggests that the blue arrow could be for -15 (or $+\widehat{15}$), agree that 30 - 15 = 15 (or 30 + $\widehat{15}$ = 15), but then ask what else the blue arrow could be for.

EN 8

When someone suggests $\frac{1}{2}$ x, write "$\frac{1}{2}$ x" in blue near the arrow picture.



T: The blue arrow is for $\frac{1}{2}$ x. What number is $\frac{1}{2}$ x 30?

S: 15.

Erase "15" and "30". Point to the dot on the <u>left</u>.

T: If this number were 13, what number would the other dot be for?

S: 26.

T (tracing the red arrow): Yes, 2 x 13 = 26.

Continue in this manner with several other starting numbers at the left dot such as: 7; 23; 500; and 1,000.

Point to the dot on the <u>right</u>.

T: If this number were 8, what number would the other dot be for?

S: 4.

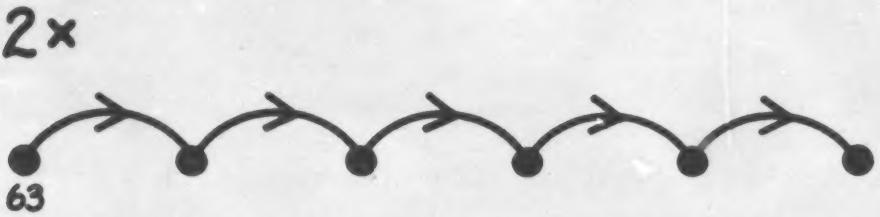
T (tracing the blue arrow): Yes, $\frac{1}{2}$ x 8 = 4.

Continue with several other starting numbers at the right dot such as: 18; 20; 62; 16; and 100.

Alternating dots, assign a number to one of the dots and ask what the other number in the picture would be. Choose numbers appropriate to the abilities of your students.

Exercise 2

Draw this arrow picture on the board.



T: What are the red arrows for?

S: 2 x.

Point to the second dot from the left.

T: Tell me with whispers what number is here.

Accept whispers from several students and then ask someone to give the answer (126) aloud.

T: How can we calculate "2 x 63"?

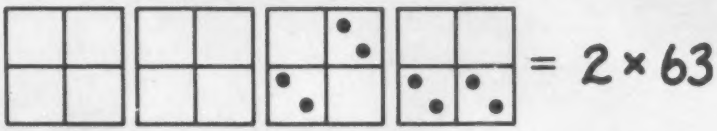
S: Use the Minicomputer.

Display four Minicomputer boards.

T: How can we show 2 x 63 on the Minicomputer?

EN 8

Guide the discussion until it is clear that 63 should be put on the Minicomputer twice, and then ask two students each to put 63 on the Minicomputer.

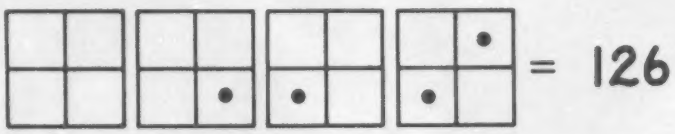


T: What could we do to make the number easier to read?

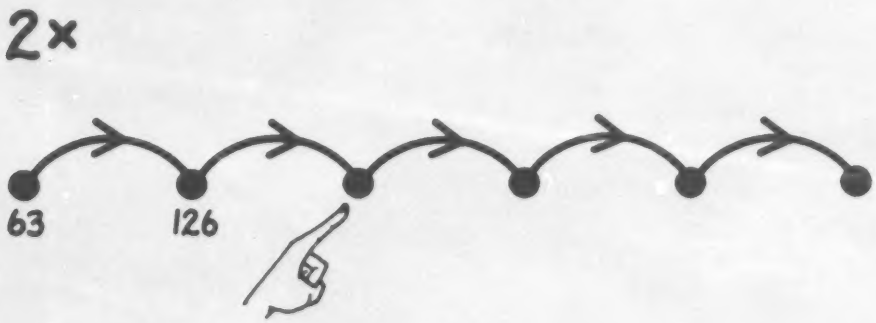
S: Make some plays.

Although the "20 + 20 = 40" play is not the most efficient, it is a correct play and should be accepted. If this play is made, later someone will need to make the "40 = 20 + 20" play in order to set up the "80 + 20 = 100" play. It may be necessary for you to suggest a backward play.

Invite students to make plays until the standard configuration for 126 is obtained.



Label the second dot "126" and point to the third dot.



T: Tell me with whispers what number is here.

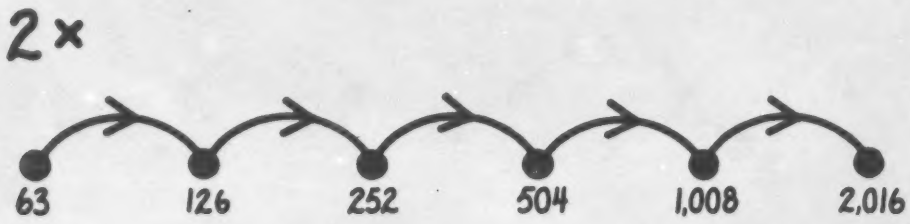
Accept many whispers, but do not be surprised if only a few of your students answer correctly.

T: How can we calculate "2 x 126"?

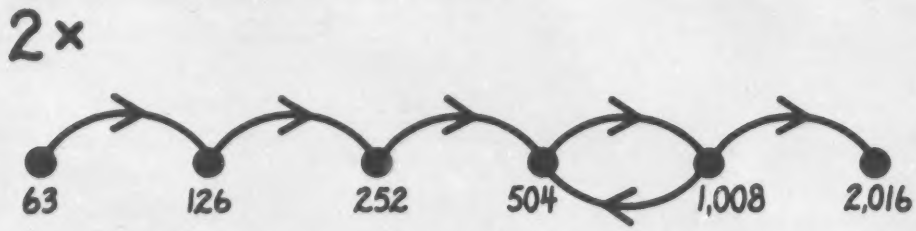
S: On the Minicomputer; put on 126 two times.

Ask two students each to put 126 on the Minicomputer. Then invite students to make plays until the standard configuration for 252 is obtained. Ask students who had whispered the correct answer to you earlier how they had done the calculation.

Continue in this manner until all the dots are labeled.



Draw a blue return arrow from 1,008 to 504.



T: What could this blue arrow be for?

EN 8

S: $\frac{1}{2}$ x.

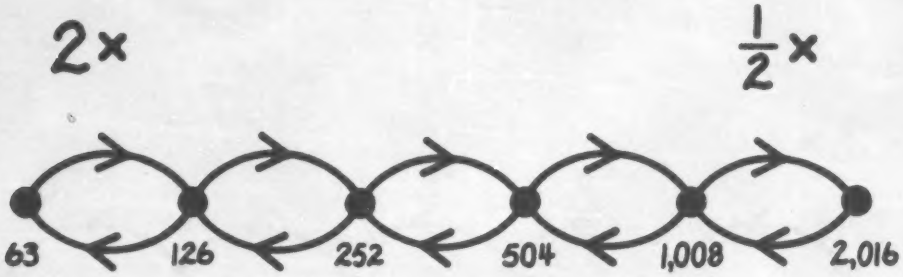
Write "$\frac{1}{2}$ x" in blue near the arrow picture.

T (tracing the blue arrow): $\frac{1}{2}$ x 1,008 = 504.

Write "$\frac{1}{2}$ x 1,008 = 504" on the board.

T: Could we draw some more blue arrows?

Invite students to draw the remaining $\frac{1}{2}$ x arrows and for each one record the appropriate number story on the board.



$\frac{1}{2}$ × 2,016 = 1,008

$\frac{1}{2}$ × 1,008 = 504

$\frac{1}{2}$ × 504 = 252

$\frac{1}{2}$ × 252 = 126

$\frac{1}{2}$ × 126 = 63

EL1 Sending Letters

CAPSULE LESSON SUMMARY

Present an arrow picture where the dots are for children and the arrows are for "sent a letter to". Collectively, discuss some of the many observations that can be made about this picture. Individually, draw similar arrow pictures.

MATERIALS

Teacher: Colored chalk
Student: Unlined paper; colored pencils

DESCRIPTION OF LESSON

Begin the lesson with a brief exchange about writing letters to friends.

T: I have a friend, Nick, who has just started third grade at another school. He gave me an arrow picture that he had drawn himself. Nick's picture is very interesting and I would like to show it to you. I will draw it on the board so everyone can see.

Draw eighteen well spaced dots on the board.

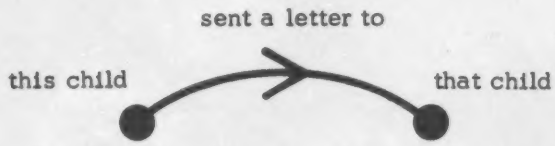
EL 1

T: These are for the children in Nick's third grade class. How many children are in Nick's class?

Call on two students one at a time to go to the board and count the dots, pointing to each dot as they count. The class should agree that there are eighteen children in Nick's class.

T: Nick told me that during the summer vacation some of the children in his class wrote letters to other children in the class. He drew red arrows to show this.

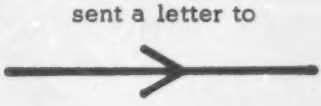
On another part of the board, draw this picture.



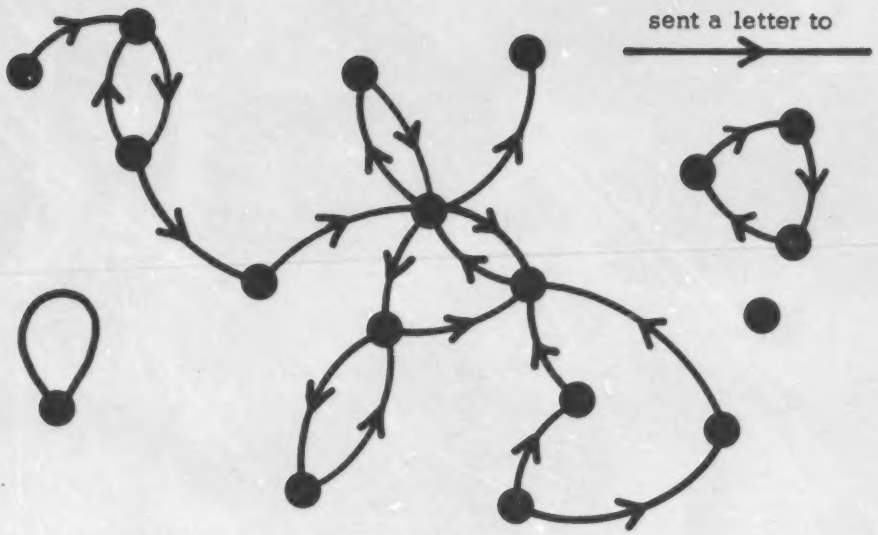
Ask a student to read the message of the red arrow as you point to the starting dot, trace the arrow, and then point to the ending dot.

S: This child sent a letter to that child.

Erase this picture and draw a key arrow near the eighteen dots.



T: Nick's picture has many red arrows so it will take me a couple minutes to finish the drawing on the board. I would like to surprise you, so everyone cover your eyes while I complete the picture.



T: You may open your eyes now. Look at the picture on the board. Think carefully about this picture for a moment without saying anything.

After about a minute, ask the class what they think about Nick's picture. Let students comment spontaneously, even if their remarks are not directly related to the meaning of the arrows. For example, the following comments might be made.

S: It's a very complicated picture.

S: There are lots of arrows.

S: It looks like a dancer.

S: I see a child who sent a lot of letters.

S: I see a child who received a lot of letters.

S: I see someone who didn't receive any letters.

When a comment like one of the last three here is made, ask the student to point to the dot at which he or she is looking and to explain. Be sure that arrows which start at a dot are counted as letters sent and that arrows which end at a dot are counted as letters received.

Let the discussion continue as long as new observations are being made. When the students stop making comments, ask some specific questions; for example,

T: Nick told me that he is the child who sent the most letters. Where is Nick? How many letters did he send? (Four)

Ask a student to point to the dot for Nick and to trace the four arrows for the letters Nick sent. Label the dot.

T: Can you find a child who sent just one letter and received just one letter?

There are many, so allow several students to point to different dots at which just one red arrow starts and just one red arrow ends.

T: Nick told me that his friend Jenna received the most letters. Where is Jenna. How many letters did she receive? (Four)

Ask a student to point to the dot for Jenna and to trace the four arrows for the letters Jenna received. Label the dot.

T (pointing to the dot with a loop): What do you think about this child?

S: That child sent a letter to herself (himself).

T: Find some children who exchanged letters; that is, sent letters to each other.

There are several such pairs of children, so allow students to find as many as they can.

T: Nick told me that there is a new pupil in his class. This child sent no letters and received no letters. Where is the new pupil in this picture?

Ask a student to point to the appropriate dot.

T: In the story told by Nick's picture, how many letters were sent in all? (Twenty-two) How many letters were received? (Twenty-two)

If you could be one of the children in Nick's class, who would you like to be? Explain why.

Students will react differently to such a question; encourage them to express themselves.

Distribute unlined paper and colored pencils. Ask the students to draw a picture for their own story about sending letters. One way to get them started is to ask them to draw dots for themselves (label it) and then to draw dots for some of their friends. Now ask them to draw some red arrows as they wish which tell a story about sending letters.

You will receive many different pictures. Show some of them to the whole class and let other students comment briefly. You may wish to display some pictures on a bulletin board.

EL 2

EL2 Introduction to A-Blocks

CAPSULE LESSON SUMMARY

Introduce the A-Blocks and describe each piece with three attributes: color, shape, and size. Find pieces which differ in one way, two ways, and three ways. Place pieces in a string picture and find pieces which belong in a certain region of the string picture.

MATERIALS

Teacher: A-Block String Game kit; colored chalk; box
Student: None

NOTE: Before this lesson begins, you will need to prepare some of the materials from the A-Block String Game kit. These materials will be used in all future lessons on A-Blocks and the String Game, so keep them together in the envelope with the kit.

Punch out one set of shapes (twenty-four pieces). If you have a magnetic chalkboard or gridboard, magnetize each piece by sticking a small piece of the magnetic material to the back. If you do not have a magnetic board available, stick a loop of masking tape to the back of each piece. Masking tape loses its stickiness quickly, so be prepared to reinforce the pieces with new loops of tape during the lesson.

DESCRIPTION OF LESSON

Put the twenty-four A-Block pieces (shapes) in a box about the size of a shoe box. Sort them so that you can locate any given one quickly.

Exercise 1

T: In this box I have some cardboard pieces in different shapes, colors, and sizes. They are called "A-Blocks". What do you think are the different shapes of the pieces?

S: Squares.

S: Triangles.

S: Circles.

You may need to tell the class the three shapes involved. If rectangles are suggested, give the clue that you have a special kind of rectangle. As each shape is mentioned, write the word on the chalkboard.

T: What are the different colors of the pieces?

S: Red.

S: Blue.

S: Green.

S: Yellow.

As each correct color is mentioned, write the word on the chalkboard. If a color other than the correct four is suggested, simply say, "No, I have no pieces that color.

T: There are two different sizes; what are they?

S: Big and little.

Write these words on the chalkboard.

Tell the class it will be their job to get the pieces out of the box. Call on students one at a time to describe to you which piece they want to see. As the pieces are described and taken out of the box, put them on the chalkboard for all to see.

EL 2

T: You tell me which piece you want, and I will take it out of the box for you.

S: A square.

T: I have many squares. Which one do you want?

S: Any of them.

Hold up a square, perhaps a big red square.

T: Here's a square. I will put this piece back in the box. Now who can tell me which piece it was?

S: A square.

T (holding up a green square): This one?

S: No, a red square.

T (holding up the little red square): This one?

S: No. It was a big red square.

T (holding up the big red square): Was it this piece?

S: Yes.

Put the piece on the chalkboard.

T: Someone else ask me for a piece.

S: A big triangle.

T: Yes, I have several big triangles. Which one do you want?

S: A big blue triangle.

Take out the big blue triangle and put it on the chalkboard.

T: Very good. Sam told me the size, the color, and the shape of the piece he wanted to see.
Can someone describe another piece with the same shape?

S: A big yellow triangle.

Put the big yellow triangle on the board and continue asking for pieces. If someone describes a piece already on the board, point to it and ask for one not yet seen. If the activity begins to drag, ask a student to describe the rest of the triangles and put them up as they are described. In this way, get all the triangular pieces on the chalkboard.

T: How many triangles do we have?

S: Eight.

T: How many circles do you think there are?

S: Eight.

Take out all (the rest of) the circles and put them on the board.

T: What else could be in the box?

S: Just some squares.

T: Which ones . . . ?

Take them out as they are described and put them on the board.

T: How many pieces are there altogether?

S: Twenty-four.

T: Can you explain?

S: I counted them.

S: Eight triangles, eight circles, and eight squares make twenty-four pieces in all.

Write this calculation on the board.

$$8+8+8=24$$

T: Can anyone tell us another number story for this?

Accept any number story for 24, but write only "3 x 8 = 24" on the board. If necessary, suggest this multiplication story yourself.

$$8+8+8=24$$

$$3\times8=24$$

Exercise 2

Take a large yellow circle and put it on a separate section of the board.

T: Who can show me a piece that is different from this piece in just one way?

Suppose a student, Jason, chooses a small green circle.

one way

T: Well, Jason, your piece is a different color but it is also a different size, so it is different from my piece in two ways.

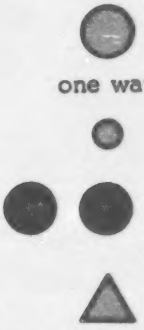
Remove the small green circle and ask the student to make another choice. Suppose Jason selects a small yellow circle.

one way

T: Is Jason's piece a different color? (No)
Is it a different shape? (No)
It is a different size? (Yes)
So the pieces are different in just one way. Who can show another piece different from mine in just one way?

Invite several students, one at a time, to select a piece. Each time let the rest of the class check that a correct piece was chosen. It is not necessary that all the pieces which differ in just one way be found; but, for your information, all the possibilities are shown in the next illustration.

EL 2



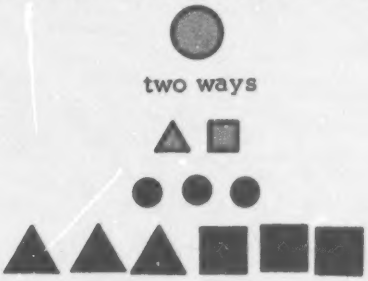
one way

(different only in size)

(different only in color)

(different only in shape)

If the lesson has been moving along at a brisk pace, the class might enjoy finding some of the pieces which differ from yours in two ways and perhaps even in three ways. All the possibilities are shown below.



two ways

(different only in size and shape)

(different only in size and color)

(different only in shape and color)

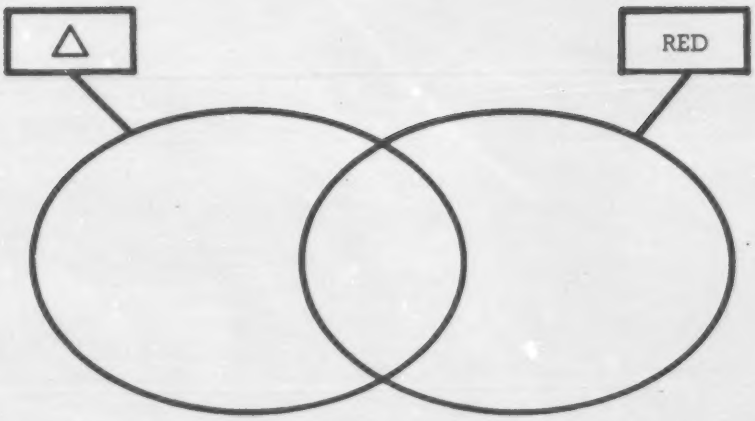
---

three ways

(different in size, shape, and color)

Exercise 3

Put all the pieces back in the box. Draw two large overlapping strings in two colors and label them as in the next illustration.



T: The blue string is for all the triangles. The red string is for all the red pieces.

Hold up the large red square.

T: Who can put this piece in the picture?

Call on a student to put the piece in the picture and ask the class for agreement or disagreement. If the piece is incorrectly placed, ask for an explanation and call on another student to place the piece. If the piece is correctly placed, ask:

T: Why do you think Kristy put it inside the red string?

S: Because the red string is for red pieces.

T: Why did Kristy put it outside the blue string?

S: Because the blue string is for triangles, not squares.

T: Yes, you must think about both strings.

T: Let's put another piece in our picture. Where should we put this little yellow circle?

Hold up the little yellow circle, and call on a student to put it in the picture. Again, ask the class for agreement or disagreement.

T: Does it go in the red string?

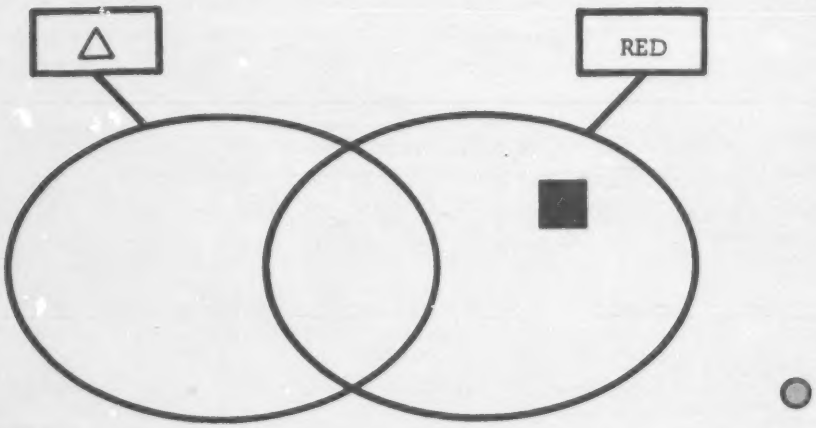
S: No, it is not red.

T: Does it go in the blue string?

S: No, it is not a triangle.

T: Then it is outside both strings.

Let a student put the piece correctly in the picture, if it has not already been done.



Continue in this way, asking students to place pieces in the picture. Not every piece needs to be discussed, but try to get at least one piece in every region. Be sure the students understand the two questions that help one decide where a piece goes: Is it red? Is it a triangle?

Sometimes vary the activity by asking:

T: Can you find a piece that belongs here?
(or)
I'll put this piece here. Is it correct?

Leave the pieces presently on the board and continue the exercise with some new types of activities.

Ask a student to describe a piece still in the box, and then invite the student to put it in the correct place in the picture. Repeat this activity several times.

Put a dot in the string picture. Ask a student to find a piece in the box that the dot could be for. Let the class decide whether or not the choice is correct. If it is correct, put that piece on the board next to the dot. Ask another student to find a piece in the box that the same dot could be for. If there are no more possibilities or a student thinks there are none, ask the student to explain. Repeat this activity with a dot in another region.

EL 3

EL3 String Game with A-Blocks

CAPSULE LESSON SUMMARY

Review the sizes, shapes, and colors of the A-Block pieces by taking them one at a time out of a box as they are described. Play the A-Block String Game.

MATERIALS

Teacher: A-Blocks String Game kit; colored chalk; magnetic board (optional)

Student: None

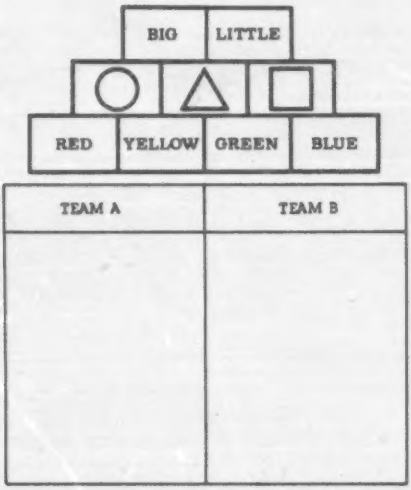
NOTE: Before this lesson begins, you need to prepare some materials from the A-Blocks String Game kit. If you use a magnetic board (chalkboard or grid board), punch out and magnetize the attribute cards from the kit. Do this in the same way as you magnetized the shapes for Lessons EL2, except in this case the magnetic material is attached to the front face of the cards so that they can be placed face down (hidden) as labels for strings. If you do not have any magnetic board available, prepare the attribute cards with loops of masking tape in a way similar to the way the shapes were prepared. Be careful not to obscure what is written on the card and be prepared to make repairs during the lesson as masking tape loses its stickiness quickly.

DESCRIPTION OF LESSON

Put the A-Block pieces (shapes) in a box about the size of a shoe box. Sort them so that each one is easy to locate. Divide the class into two teams and try to arrange that the teams are roughly equal in ability.

Exercise 1

Prepare a team board as shown in the next illustration and post a list of attributes above it. Briefly review the different kinds of A-Block pieces.



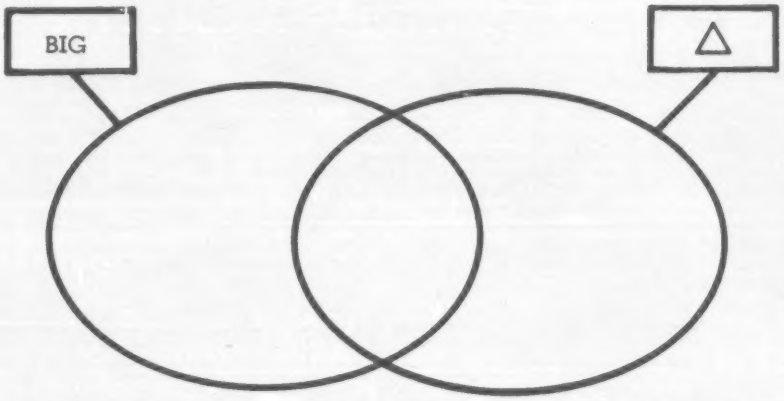
T: We are going to play a String Game with these shapes. First you have to help me take the A-Block pieces out of this box. We will take turns; someone on Team A will tell me which piece to put on Team A's side of the board, and then someone from Team B will tell me which piece to put on Team B's side of the board. Remember, you must describe a piece that is still in the box and so that I know exactly which piece you want me to put on your side of the board.

Alternating between the teams, let students describe pieces to put on the team board. Insist that the description be complete. If the piece described is already on the board, point it out and ask for another. Try to call on weak students early so that there are many choices open to them. Encourage all the students to be thinking about which pieces they will ask for when you call on them.

EL 3

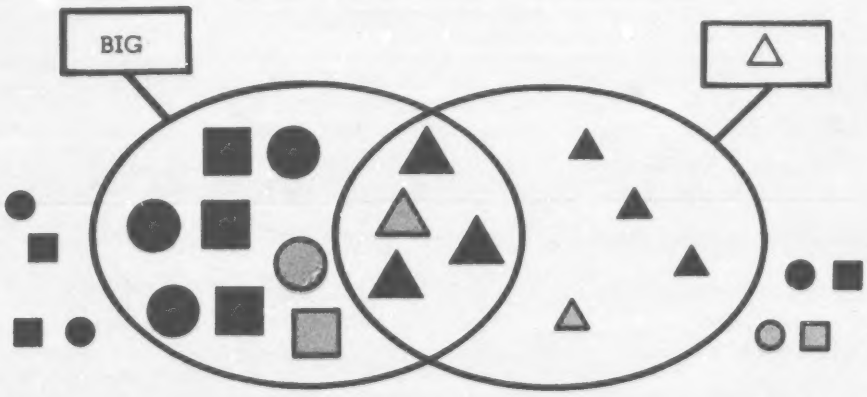
Exercise 2

Draw two large overlapping strings on the playing board using different colors. Label the strings as in the next illustration.



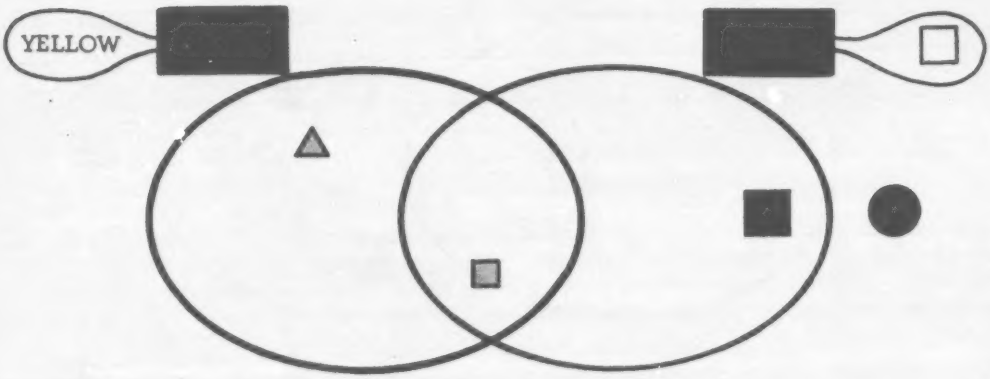
T: Now the teams will take turns trying to place their pieces in the string picture. Each of you will try to place just one piece from your team's side of the board. If you place it correctly, I will say "Yes." If you put it in the wrong place, I will say "No" and you must return the piece to your team's side of the board. The first team to get all of its pieces in the string picture is the winner.

Play this game alternating teams and letting the members of a team take turns. The picture below shows all of the A-Block pieces placed correctly.



Exercise 3

Prepare to play the String game in the usual way with face down string cards.* Redistribute game pieces on the two sides (TEAM A and TEAM B) of the team board. Set up the playing board as shown below. Bubbles indicate what is hidden on the string cards. Four correctly placed pieces are the starting clues.



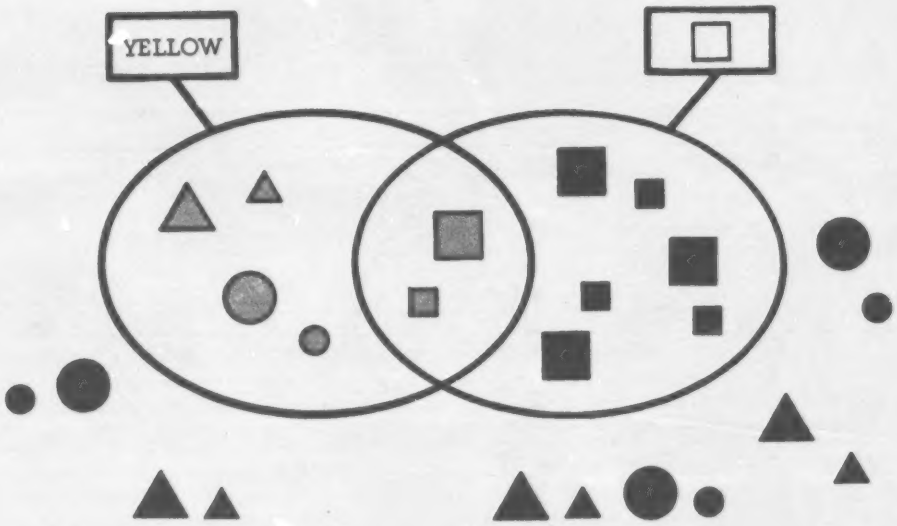
Remember that the rules of the game call for you to be the judge. If a piece is correctly placed, say "Yes" and immediately instruct the player to try to place a second piece. No player should make more than two consecutive plays. If a piece is incorrectly placed, say "No" and ask the player to return the piece to his or her team's side of the team board.

For this game, the player who correctly places the last piece from his or her team's side of the team board may then attempt to identify each of the string cards. A team wins by being first to get all of its pieces in the string picture and to identify the strings.

* See APPENDIX: The String Game at the end of The Languages of Strings and Arrows strand in UPG-III for a description of equipment and preparation for the game played in the usual way.

EL 3

The following picture shows all of the A-Blocks placed correctly. You could use this picture as a "crib-sheet" while the game is being played.



EL4 Multiples

CAPSULE LESSON SUMMARY

Play a type of "Guess My Rule" game where the players decide that a given string is for multiples of 3. Locate many integers, positive and negative, in the string picture. Label some dots in a double spiral arrow picture and find that the arrows could be for +3. Then observe that the numbers on the spiral are the same as those inside the string. Discuss other multiples, for example, multiples of 5 and of 2. Locate numbers in a two-string picture with one string for multiples of 2 and the other for multiples of 5.

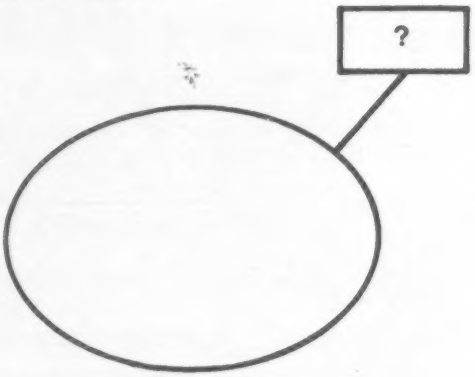
MATERIALS

Teacher: Colored chalk
Student: Paper; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Draw a large string on the board.



NOTE: The string is for multiples of 3, but do not announce this or use the word "multiples" until later in the lesson.

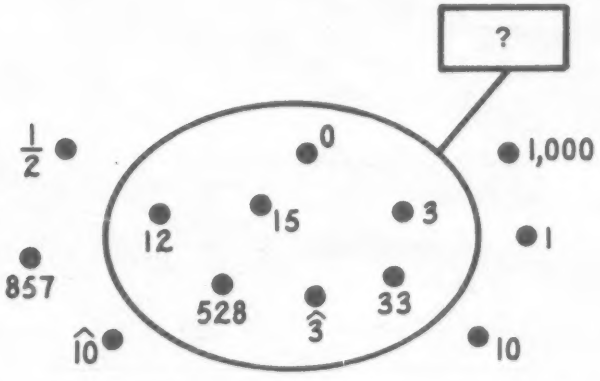
Tell the class that you are not going to tell them yet what this string is for except that it is for some numbers. They may choose numbers and you will place

them correctly in the picture, either inside the string or outside the string. When they think they know what the string is for, they may guess and you will tell them whether or not they are correct.

Give each student a chance to choose a number or to guess what the string is for. That is, go around the room and ask each student, in turn, either to give you a number to put in the picture or to guess what the string is for.

NOTE: For your information only, an integer is a multiple of 3 if and only if the sum of its digits is a multiple of 3.

After a short while your picture should have many numbers in it and some students may have made some "wild" guesses. For example,



If students guess that the string is for even numbers or odd numbers or more than 10 or less than 1,000 or something else which can easily be seen to be impossible, ask the class to say why the guess is incorrect. For example,

S: The blue string cannot be for even numbers, because 15 is inside and 15 is odd.

S: The blue string cannot be for less than 1,000, because 10 is outside and 10 is less than 1,000.

Although your students may be unfamiliar with the term "multiples of 3", they soon should begin to notice that the numbers inside the string have something to do with counting by threes. When this occurs, invite students to predict where certain numbers belong in the picture.

T: Where should I put 30?

S: Inside the string, because you say "30" when you count by threes.

Invite a student to count by threes starting at 0 and ending at 30.

T: Where should I put 37?

S: Outside the string, because from 30 we go next to 33, 36, 39 and skip over 37.

T: Where should I put 300?

S: Inside the string, because if you count 100 times by threes you'll get to 300.

If you already have some negative numbers in the picture, discuss also counting backward by threes and put some more negative numbers in the picture. Otherwise, wait for the next exercise to discuss negative numbers.

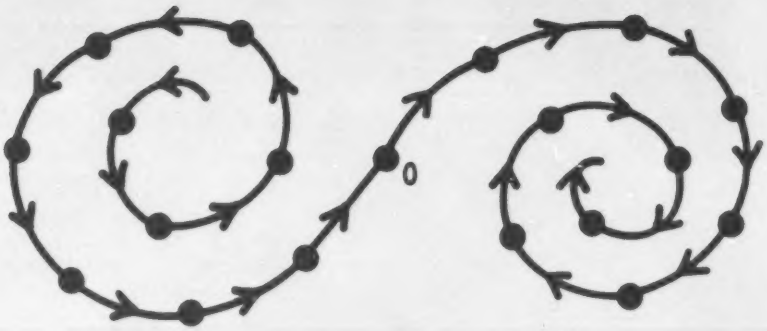
T: We have a name for the numbers inside this string. They are called "multiples of 3".

Write this term inside the label for the string. Leave your picture on the board as you will refer back to it in Exercise 2.

Exercise 2

Draw a double spiral arrow picture on the board and label the middle dot "0". (See the illustration on the next page.)

EL 4

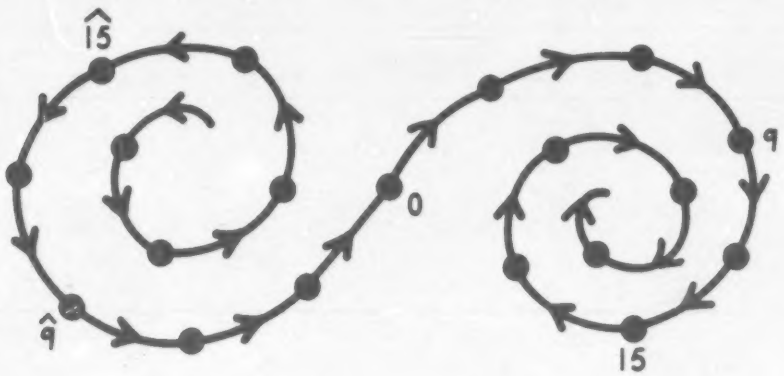


T: What do you think about this picture?

Let the class react. Perhaps they will mention its symmetry, that 0 is in the center, that it could go on and on in both directions, etc. Allow a minute or so for such observations before continuing, but do not insist that they mention these things.

T: Let's put some more numbers in the picture.

Label dots for 9, $\hat{9}$, 15, and $\hat{15}$.

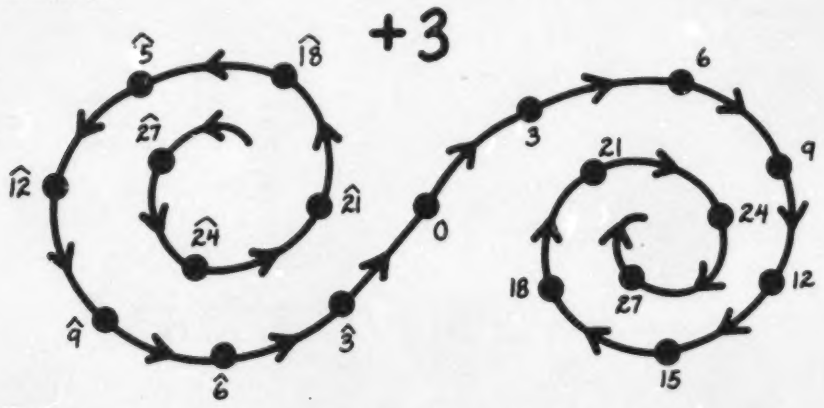


T: What could the red arrows be for?

Allow the students to experiment with some possibilities. For example, if a

student suggests +9, observe that then 9 would be at the end of the arrow starting at 0, not where it is. Perhaps a student will suggest "is less than" and then you could observe that this is a possibility, but ask for another possibility. When a student says "+3", check that indeed the red arrows could be for +3 and write the key near the picture. Otherwise, put in some more numbers, say 6 and $\hat{6}$, and ask the question again. After someone suggests that the red arrows are for +3, write the key on the board.

Invite students to label dots in the picture. Some of your students may have trouble labeling the dots in the part of the spiral to the left of 0. Help them by referring to the number line, to Eli's peanuts, or to the floors in the Empire State Building.



T: What do you notice about the numbers in this picture?

S: They are alike on both sides; 3 and $\hat{3}$, 6 and $\hat{6}$, 9 and $\hat{9}$, and so on.

S: They are the numbers we say when we count by threes, forward and backward.

S: They are the numbers inside the string, the multiples of 3.

If this last observation is not made call the class' attention back to the string picture and ask specifically which numbers in the string picture would be in

the arrow picture. (The numbers inside the string)

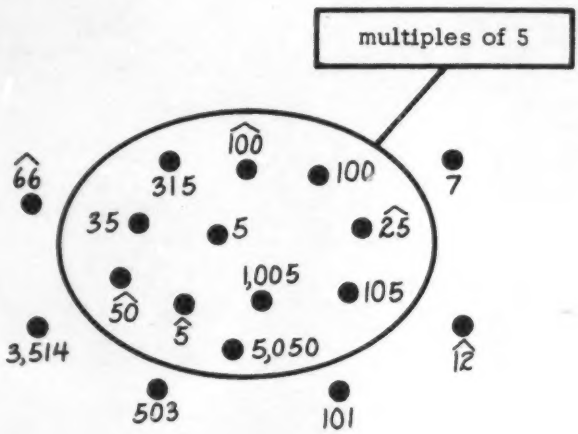
Continue by asking students to predict whether certain numbers are multiples of 3; that is, whether they would be in the double spiral for +3. Include some large numbers such as 300, 333, 900, 906, and 3,000.

At this point, if there are no negative numbers in the string picture, direct the class to observe that all the numbers in the arrow picture are inside the string, including those which are negative numbers.

Exercise 3

Repeat Exercise 1 for multiples of 5. The students should quickly guess that your new string is for multiples of 5 and perhaps will even use the expression "multiples of 5".

After the class discovers that your string this time is for multiples of 5, let them put in some numbers themselves. Encourage students to include both positive and negative numbers and some large numbers.

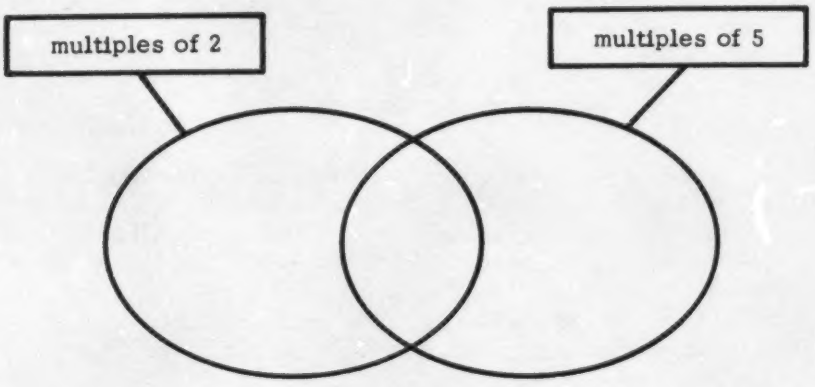


T: If I were to draw an arrow picture as before to include all the multiples of 5, what kind of arrows would I use?

S: +5.

Exercise 4

Ask the class for numbers which are multiples of 2 (even numbers) and then draw this picture on the board.



Instruct the students to draw this picture on their papers and to put in as many numbers as they can.

EL 5

EL5 Project: Making a Temperature Bar Graph/The String Game

CAPSULE LESSON SUMMARY

Introduce a long term project of keeping a record of the morning temperature in degrees celsius. Play the String Game with A-Blocks. Also play a type of string game with numbers where the strings are for multiples of 3 and multiples of 5.

MATERIALS

Teacher: Thermometer; grid paper; Language Poster #2 for UPG II; String Game with A-Blocks kit; colored chalk

Student: None

NOTE: For this lesson you will need to obtain an outdoor celsius thermometer which can be mounted outside of a classroom window and read from inside the room. This thermometer should be calibrated each degree and labeled at least every 10 degrees. Post one of the charts from the poster on a bulletin board or some other convenient place in your classroom. Your class will record daily temperatures on this chart for the next two months, so it should be located in a place where it can stay during that time.

DESCRIPTION OF LESSON

On the morning that this lesson is taught (preferably the first class day in January), make note of the temperature in degrees celsius.

Begin the lesson by showing the class the thermometer and allow the students a few minutes to examine it close up. Discuss how one reads the temperature on a thermometer, perhaps reading and calling on individual students to read the (indoor) classroom temperature on this thermometer. Make special note to tell them that the scale here is for degrees <u>celsius</u> (write this word on the board) and note the classroom temperature as (for example) 22° C. Discuss the fact that sometimes the temperature is above zero and sometimes below zero. For a temperature below zero, for example five below zero, we write "$\hat{5}$° C".

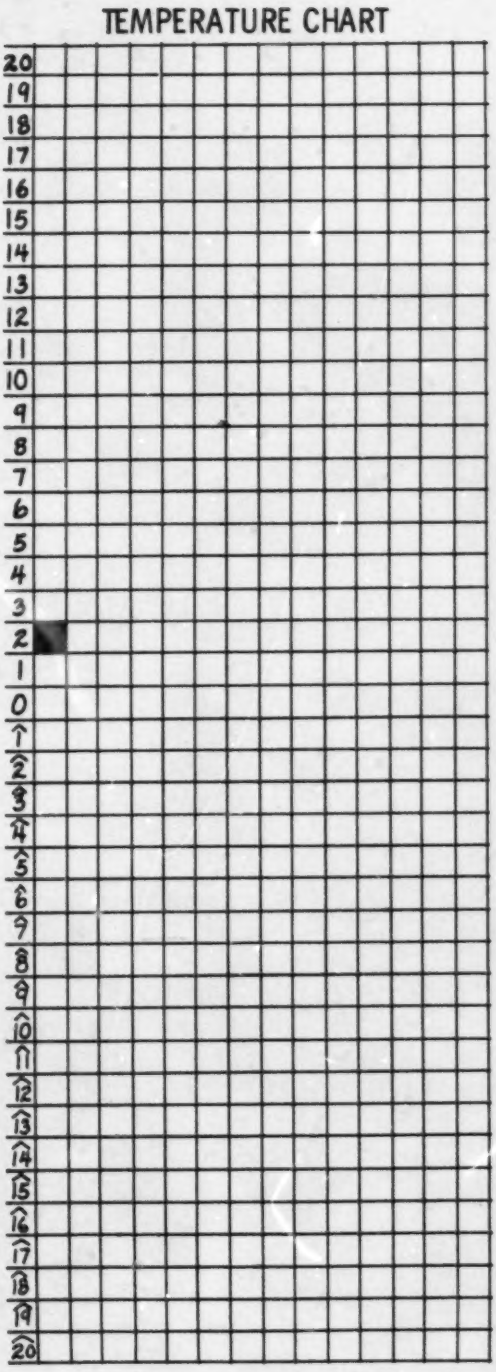
T: We are going to put this thermometer outside our classroom window, and every morning* we will record the temperature on this chart.

This morning I read my outdoor thermometer at home so we could record today's morning temperature. It was (for example) 2° C. Where do you think we should record this on our chart?

Invite a student to locate (in this case) 2 (above zero) on the chart.

T: To record the temperature, we shade the square next to "2".

Ask a student to do this as indicated in the chart here.



* It would be best to read and record the temperature at the same time every day; we suggest first thing each morning.

EL 5

NOTES:

1. Your class will be keeping a record of the morning temperature in degrees celsius for the next two months (school days in January and February). Let different students read the thermometer and record the temperature each day. Every student should have at least one or two chances to do this.

2. The first time a student reads the thermometer, check it with him or her. If the mercury is between two marks, read the temperature to the nearest degree.

3. Each day you might ask one or two questions. For example,
   a) What do you think the temperature will be this morning? After taking the temperature, ask who had the closest guess.
   b) Yesterday the temperature was 2° C and today it is $\hat{3}$° C. Is it warmer or colder today? How many degrees colder?
   c) What is the coldest temperature we have recorded so far? I heard that this morning was the coldest morning of the year. What do you think the temperature is this morning?

For the rest of the lesson do the exercises from L13 The String Game #3, UPG III.

EL6 Temperature Bar Graphs

CAPSULE LESSON SUMMARY

Make temperature bar graphs for some given temperature data from two different cities. Compare these graphs to the temperature bar graph your class has been making during the past two months.

MATERIALS

Teacher: Classroom temperature chart (see Lesson EL5); Language Poster #2 for UPG-II; felt-tip marker

Student: "Temperature data and chart" sheet; pencil

DESCRIPTION OF LESSON

Begin the lesson by discussing the classroom temperature chart on which you have been recording daily temperatures during the months of January and February. The chart should be a bar graph for your data similar to those at the end of this lesson. Some questions which you might ask during this discussion are:

- During the past two months, has the temperature usually been above 0° C or below 0° C?
- What is the highest temperature we have recorded?
- What is the lowest temperature we have recorded?
- What is the difference between the highest and lowest temperature?
- Is there any temperature between the high and the low which we did not record?
- What temperature did we record most often?

Display two demonstration temperature charts (blank) from Language Poster #2. Distribute copies of temperature charts and two months of temperature data each for St. Louis and New Orleans. Explain to the class that this is temperature data collected by other classes. They are going to use this data to make bar

graphs similar to the one the class has made during January and February (point to the classroom temperature graph).

Together with the class, record for each city the first two or three temperatures in January on the blank charts you have displayed. Ask the students to do this on their papers. Then, let the students work independently recording temperatures and finishing a temperature bar graph for one or both cities. Monitor the work so that students who are having difficulty can get started correctly.

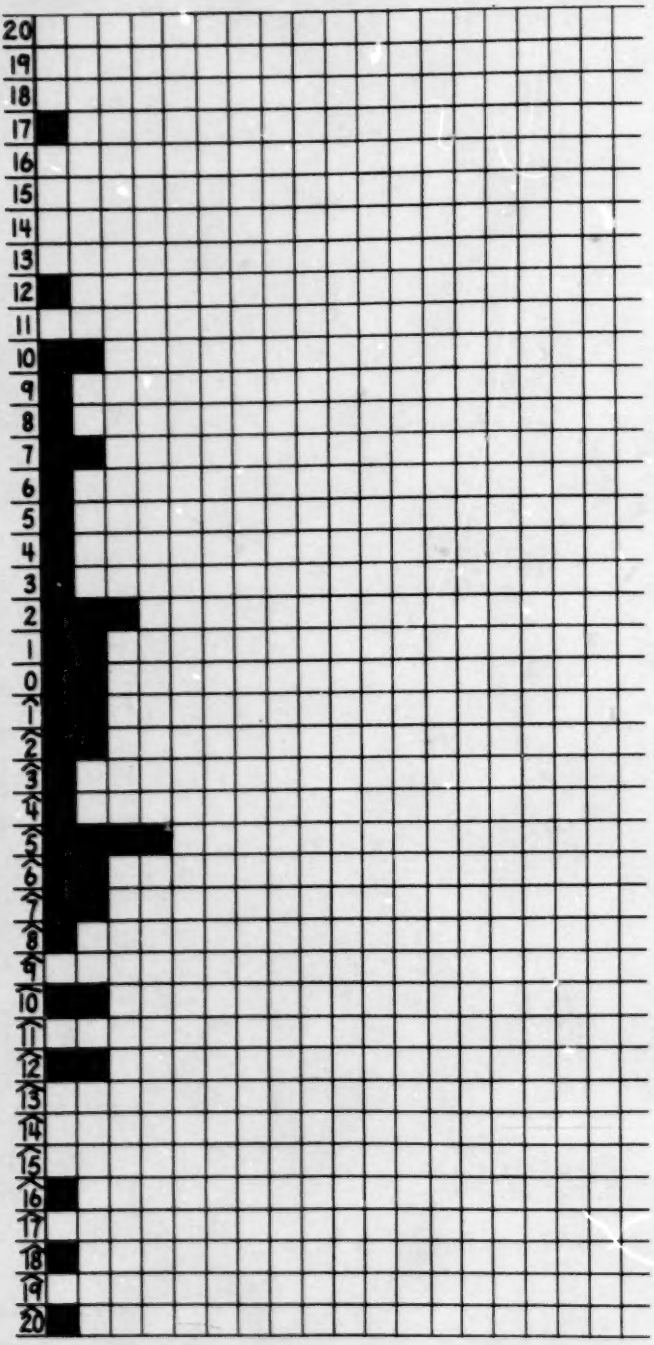
As the students are working on their own graphs, ask them one at a time to go to the demonstration charts and record some temperatures. For example, you could ask one student to record the temperatures in St. Louis for the first week in January. Then ask another student to record the temperatures in St. Louis for the second week in January; and so on.

After about 20 to 25 minutes, most students should have completed at least one bar graph, and the class should have completed bar graphs for both cities at the front of the classroom. (Following this lesson description, you will find completed bar graphs for the two sets of data included here.) Compare your classroom temperature graph, the one the class made during the months of January and February, with the ones made during this lesson. You could ask some of the same questions that you asked at the beginning of the lesson. To prompt discussion ask the following questions:

- Which city (your city or St. Louis or New Orleans) had the warmest weather in January and February?
- Which city had the warmest day in January and February?
- Which city had the coldest day in January and February?
- Which city had the greatest variation in temperatures?

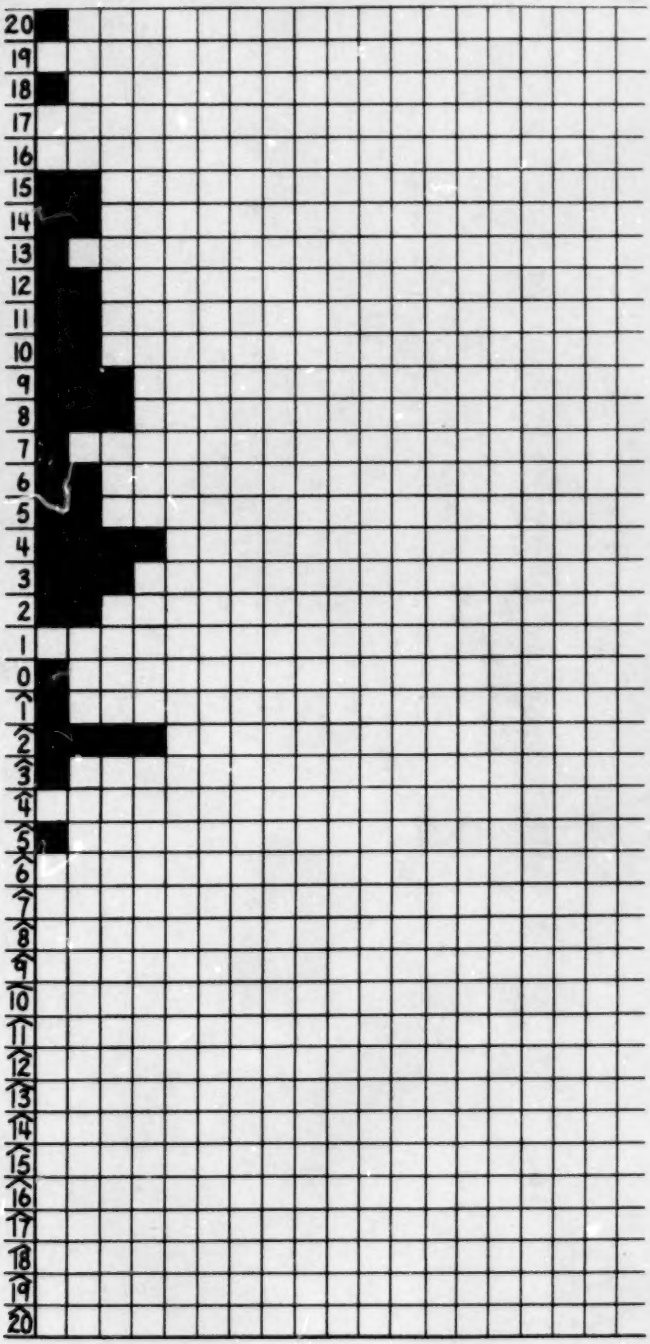
If your class has enjoyed this project, you may wish to continue taking and recording the temperature every day during March and April. In this case, you could compare your temperature graph for January and February to your temperature graph for March and April.

Temperature Graph for St. Louis, January and February 1977



EL 6

Temperature Graph for New Orleans, January and February 1977



EW1 Detective Story #1 / Introduction to Eli's Magic Peanuts

CAPSULE LESSON SUMMARY

Solve a detective story in which the secret number is an odd number and in which the other clues involve an arrow picture and the Minicomputer. Tell the story of Eli the Elephant who discovers magic peanuts; when a magic peanut meets a regular peanut, both disappear much to Eli's confusion.

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: Paper; pencil; Worksheets EW1* and **[*]

DESCRIPTION OF LESSON

Exercise 1: Detective Story

Before the lesson, write a large "21" on a slip of paper and fold the paper so that the "21" is hidden.

Ask the class what a detective does. Guide the discussion to include the idea that a detective tries to uncover secrets by following clues. Tell the students that they are going to be detectives today, and that they will be able to discover a secret number if they follow the clues.

T: The name of a secret number is written on this paper. Listen carefully to my story and you can discover what it is. The first clue I will give you is that the secret number is an odd number. What are some odd numbers?

You may need to explain odd and even numbers briefly. When most of the class understands what an odd number is, begin listing the positive odd numbers in order on the board with the students' assistance. Start with 1 and continue to

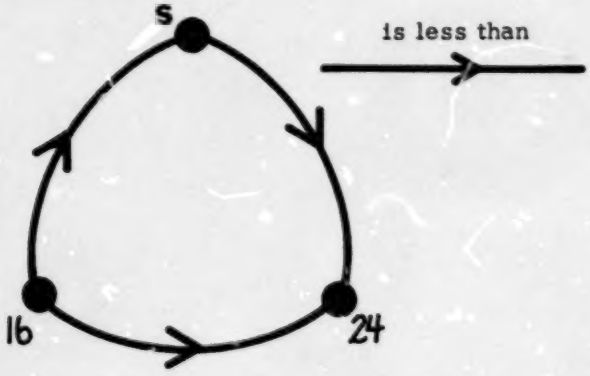
[*] These worksheets are pictured on pages 107 and 108, but are not provided for you. See the comments at the end of this lesson.

at least 25. Mention that you could continue writing odd numbers, but that instead you will put three dots at the end of your list meaning "and so on".

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27,...

T: Now we know that the secret number is one of these numbers, but we do not know which one. We need another clue.

Draw this arrow picture on the board.

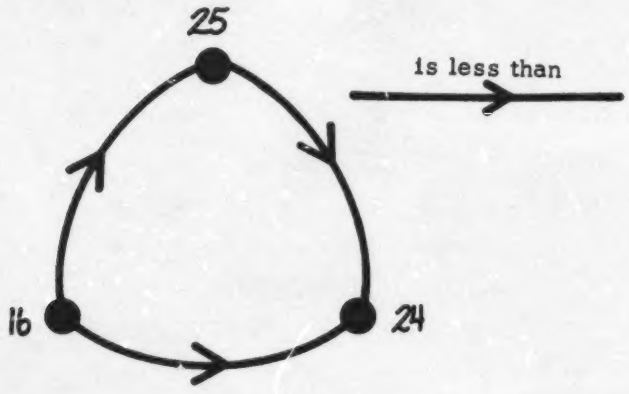


T: The secret number is here (point to "s") in this arrow picture. What are the red arrows for?

S: Is less than.

Point to the dot labeled "s" and remind the class that the secret number is an odd number. Choose an odd number between 0 and 26 and ask if it could be the secret number. If the class disagrees on whether a number could be the secret number or not, put that number in the arrow picture and ask a student to read the number story told by each arrow and its starting and ending numbers.

For example, suppose the class is not sure whether s could be 25 or not.



Trace the arrow from 16 to 25.

S: 16 is less than 25.

T: Is that true?

S: Yes.

Trace the arrow from 16 to 24.

S: 16 is less than 24.

T: Is that true?

S: Yes.

Trace the arrow from 25 to 24.

S: 25 is less than 24.

T: Is that true?

S: No.

EW 1

T: So our secret number cannot be 25.

Erase "25" from the arrow picture and cross it out in the list of odd numbers.

As the class discovers which numbers in the list of odd numbers cannot be the secret number, cross them out. Continue until every odd number in your list has been considered. Ask if there are any other odd numbers which could be the secret number. Commend a student who concludes that none of the numbers (indicated by the three dots at the end of the list) could be the secret number because they are all more than 24. If necessary, supply this information yourself and cross out the three dots at the end of the list.

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, . . .

Erase everything which has been crossed out. Display three Minicomputer boards.

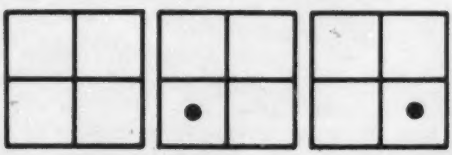
T: Now we know that the secret number is 17 or 19 or 21 or 23. Listen carefully because I will give you only one more clue. The secret number can be put on the Minicomputer using exactly two checkers.

Ask the students to write the secret number on a piece of paper and check a few of their answers.

T: Which of these four numbers is the secret number?

S: 21.

Invite a student to put 21 on the Minicomputer using two checkers.



Ask how many checkers would be needed to put each of the numbers 17, 19, and 23 on the Minicomputer. Show the class that "21" is written on your paper.

Exercise 2: Eli's Magic Peanuts

T: There is an elephant named "Eli". Eli is always very hungry. What do you think is Eli's favorite food?

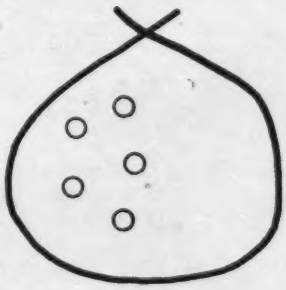
Accept some answers from the class and then tell them that Eli's favorite food is peanuts.

T: Eli likes peanuts so much that he always keeps a little bag of peanuts with him wherever he goes. One day as Eli is walking through the jungle, he finds a plant that looks a lot like a peanut plant, but not exactly like a peanut plant. Eli pulls up this strange plant and discovers five peanuts. Eli doesn't know it but these are magic peanuts. He picks those five peanuts and puts them in his bag which has five regular peanuts in it. What do you think happens?

Discuss possibilities, encouraging the students to use their imaginations.

T: Whenever a magic peanut comes near a regular peanut, it does something very strange. All of a sudden both peanuts magically disappear.

Draw this picture on the board using white chalk.

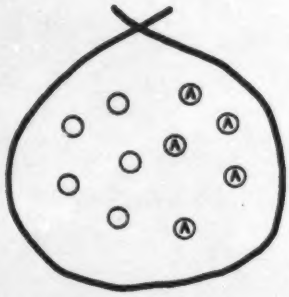


T: This is Eli's bag of peanuts with five regular peanuts in it. How many magic peanuts did Eli put in his bag?

EW 1

S: Five.

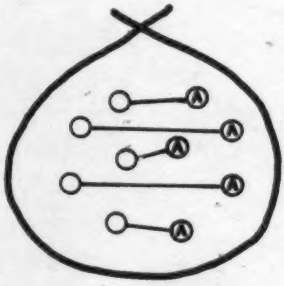
Draw five " Ⓐ " for the magic peanuts using white chalk, saying, "These are for the magic peanuts."



T: When Eli arrives home he is really hungry from walking all day in the jungle. He decides to eat some peanuts. When he takes out his bag of peanuts, he has a big surprise waiting for him. How many peanuts do you think he finds?

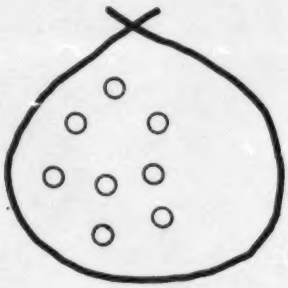
S: None.

Ask several students to explain their answers. Be sure the class understands that all the peanuts will have disappeared because there was one magic peanut for every regular peanut. Draw connecting lines between the regular and magic peanuts to show how they could be paired up.



T: Poor Eli! He is very puzzled. He doesn't know what we know about magic peanuts. He can't figure out where his peanuts have gone.

Erase the board and draw this picture.



T: The next day Eli goes out to pick peanuts. Here is his bag of peanuts. How many regular peanuts are in it?

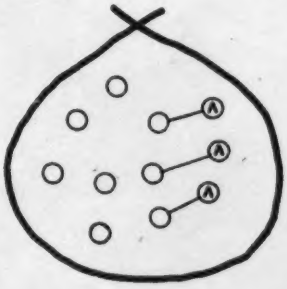
S: Eight.

T: Then Eli puts three magic peanuts into his bag. What happens?

S: The three magic peanuts and three of the regular peanuts disappear.

S: There will be three peanuts fewer in the bag than what Eli started with.

Draw the magic peanuts and lines connecting the magic peanuts with three of the regular peanuts.



T: Now we are going to write the calculation for this story. How many regular peanuts did Eli have?

S: Eight.

EW 1

Write "8" on the board.

T: And how many magic peanuts did he put in the bag?

S: Three.

T: To show that they are three magic peanuts, I'll draw a hat on the "3".

$$8 + \hat{3}$$

T: When Eli opens the bag, how many peanuts does he see?

S: Five.

T: Regular or magic?

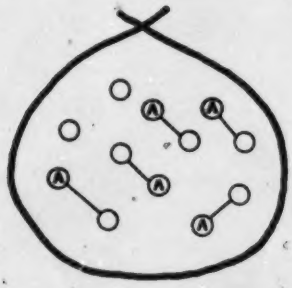
S: Regular.

T: So having eight regular peanuts and three magic peanuts is the same as having five regular peanuts in the bag.

Complete the number story on the board.

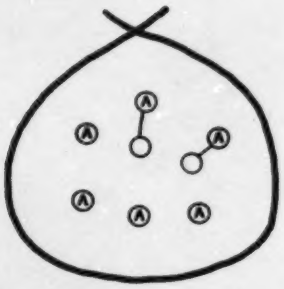
$$8 + \hat{3} = 5$$

Now draw a new picture on the board and ask the same series of questions. Use the following picture or make up one of your own.



$$7 + \hat{5} = 2$$

Do another example in which Eli finds magic peanuts when he opens the bag. For example,

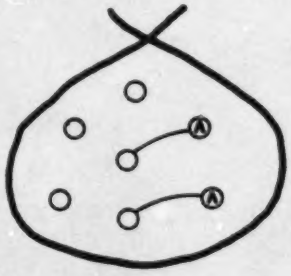


$\hat{6} + 2 = \hat{4}$

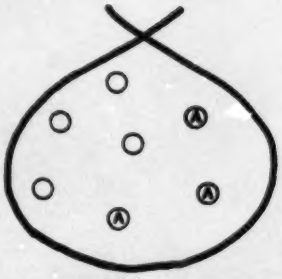
We suggest that you reproduce Worksheets EW1* and ** on pages 107 and 108 and that you use them with your class for individual work. Tell the students to finish the calculations for each of Eli's bags of peanuts. The students should draw lines connecting regular and magic peanuts themselves. Ask students who finish quickly to draw their own stories about Eli's magic peanuts.

Name _____ EW 1 *

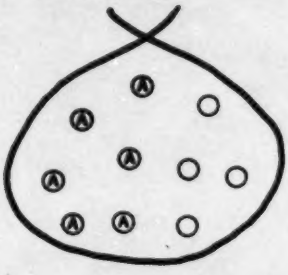
Complete these number stories.



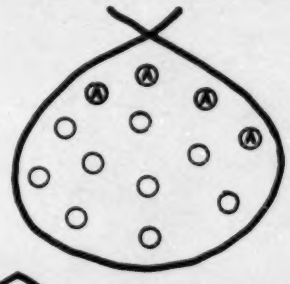
$5 + \hat{2} =$ _____



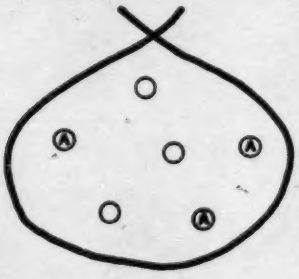
$4 + \hat{3} =$ _____



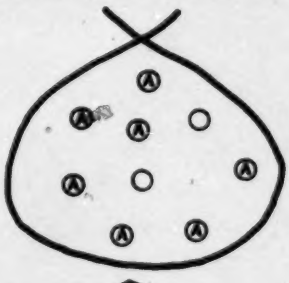
$\hat{6} + 4 =$ _____



$\hat{4} + 9 =$ _____



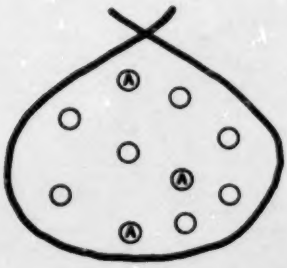
$3 + \hat{3} =$ _____



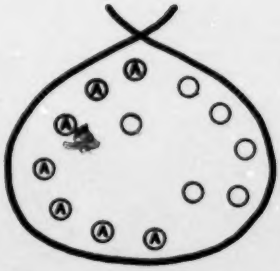
$2 + \hat{7} =$ _____

Name _____ **EW 1** **

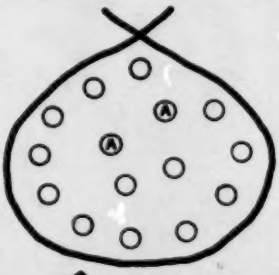
Complete these number stories.



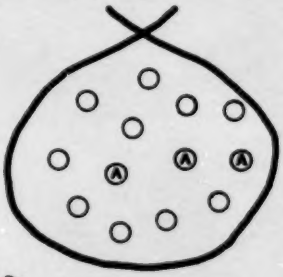
$7 + \hat{3} =$ _____



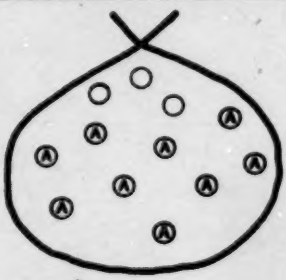
$\hat{7} + 6 =$ _____



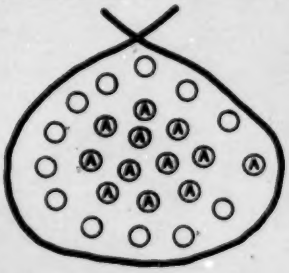
$13 + \hat{2} =$ _____



$\hat{3} + 10 =$ _____



$3 + \hat{9} =$ _____



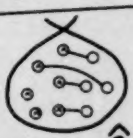
$\hat{12} + 12 =$ _____

Name _____ EW1 *

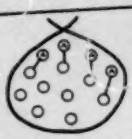
Complete these number stories.

5 + ☐ = 3

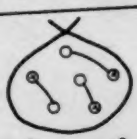
4 + ☐ = 1



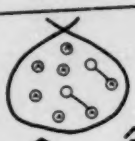
☐ + 4 = 2



☐ + 9 = 5



3 + ☐ = 0



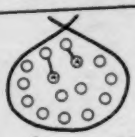
2 + ☐ = 5

Name _____ EW1 **

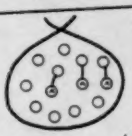
Complete these number stories.

7 + ☐ = 4

9 + 6 = ☐



13 + ☐ = 11



☐ + 10 = 7

3 + ☐ = 6

☐ + 12 = 0

EW 2

EW2 Eli the Elephant/Detective Story #2

CAPSULE LESSON SUMMARY

Introduce negative checkers to represent the number of magic peanuts in Eli's bag. Solve a detective story with clues involving the hand-calculator, a string picture, and the Minicomputer.

MATERIALS

Teacher: Two hand-calculators*; Minicomputer set; colored chalk
Student: None

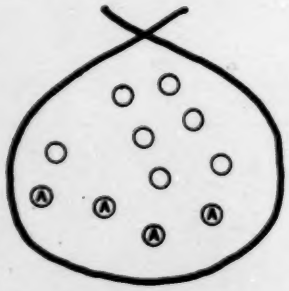
DESCRIPTION OF LESSON

Exercise 1: Eli the Elephant

Invite students to tell whatever they remember about Eli the Elephant. Review what happens when a magic peanut and a regular peanut meet.

T: Today Eli is walking in the park and finds seven regular peanuts and four magic peanuts. He puts all the peanuts in his bag. He plans to eat the peanuts tonight while he watches television. How many peanuts will he find when he opens his bag?

Draw a picture of the bag with regular and magic peanuts in it.



* See NOTE ON THE USE OF HAND-CALCULATORS in the General Introduction of UPG III.

Write a number story for this situation on the board.

$7 + \hat{4} = ?$

T: Seven regular peanuts and four magic peanuts is the same as . . . ?

S: Three regular peanuts.

Complete the number story. Display two Minicomputer boards.

T: Let's show this story about Eli's peanuts on the Minicomputer. How many regular peanuts did Eli find?

S: Seven.

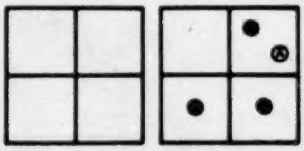
Invite someone to put 7 on the Minicomputer. Show the class the negative checkers " Ⓐ ".

T: We can use these negative checkers to show how many magic peanuts Eli found in the park. How many magic peanuts did he find?

S: Four.

T: Who can show this on the Minicomputer using negative checkers?

Do not allow the students to remove any checkers from the Minicomputer.

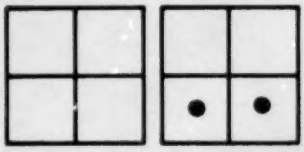


T: Now what can we do?

S: Remove the regular checker and the magic checker on the purple square.

EW 2

Explain that four magic peanuts and four regular peanuts disappear and ask a volunteer to show the class what happens on the Minicomputer. The student should remove both checkers from the purple square.



T: Seven regular peanuts and four magic peanuts is the same as . . . ?

S: Three regular peanuts.

Erase the board and remove the checkers from the Minicomputer.

T: Last week Eli found a lot of peanuts. One day he found 39 magic peanuts and 25 regular peanuts. When Eli opened his bag that night, what kind of peanuts did he find?

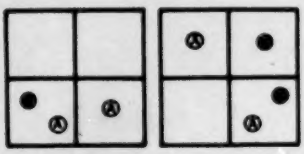
S: Magic peanuts.

Write a number story for this situation on the board.

$\widehat{39} + 25 = ?$

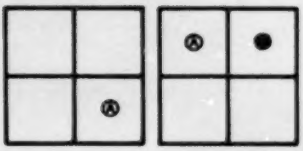
If several students know the answer, ask them to tell you the number with whispers.

T: Who can put $\widehat{39}$ on the Minicomputer with negative checkers?
Who can put 25 on the Minicomputer with regular checkers?



S: We can take off the checkers on the 1-square and the checkers on the 20-square.

Invite students, one at a time, to remove the checkers from the 1-square and then from the 20-square. Encourage students to explain that one regular and one magic peanut disappear and that twenty regular and twenty magic peanuts disappear.



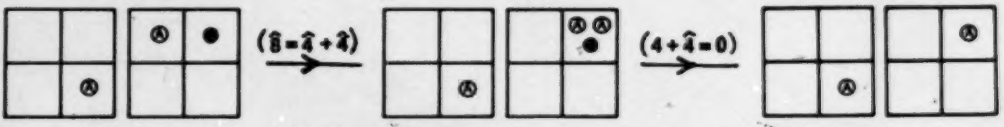
T: What could we do to make the number easier to read?

S: Make a backward play.

Do not expect the verbalization of this play to be exact. If a student says "8 = 4 + 4," accept it, but rephrase it by saying that eight magic peanuts is the same as four magic peanuts plus four magic peanuts.

If a student suggests that the negative checker on the 8-square and the regular checker on the 4-square can be taken off, remind the class that those checkers are for eight magic peanuts and four regular peanuts.

If no one suggests a backward play, suggest it yourself.



Complete the number story on the board.

$$\hat{39} + 25 = \hat{14}$$

EW 2

Repeat this exercise with "$\hat{46}$ + 22 = ?". Erase the board and clear the Mini-computer before going to Exercise 2.

Exercise 2: Detective Story

Since this is the first lesson using hand-calculators, you may want to give calculators to students who have had some experience with them before, either at home or at school. Slowly and clearly announce each step on the calculator for the entire class. The students with calculators and those close enough to observe the display of the calculator can verify what number is on the display.

Write a large "32" on a slip of paper and fold it so that the "32" is hidden.

T: The name of a secret number is written on this paper. Listen carefully to the clues.

First clue

T: The calculators will help us with the first clue. We are going to press some buttons; I will tell you which ones. One of the numbers which will appear on the calculator's display is the secret number.

Ask students with calculators to turn them "on" and remind these students to do exactly what you say. Give the following instructions.

T: Start with 0 on the display. Press [2]. What number is on the display?

S: 2.

Record "2" on the board.

T: Press [+] [5] [=].
What number is on the display?

S: 7.

T: What did the calculator do?

S: It added 5 to 2; 2 + 5 = 7.

Record "7" on the board.

T: Press [=] again. What number is on the display?

S: 12.

T: What did the calculator do?

S: Added 5 again; 7 + 5 = 12.

Record "12" on the board.

T: We can add 5 again by pressing [=] again.
Press [=].
What number is on the display?

S: 17.

Continue your list on the board.

2, 7, 12, 17

T: Suppose we continue pressing [=]. What are the next three numbers we will get on the calculator?

Allow the class to predict these numbers. Then record them on the board as the students press [=] three more times and note them on their displays. As soon as most of your students are able to predict the sequence quickly you can abandon the calculator. Continue counting and recording until the class realizes this sequence goes on and on. Your list on the board should have several numbers

greater than 50.

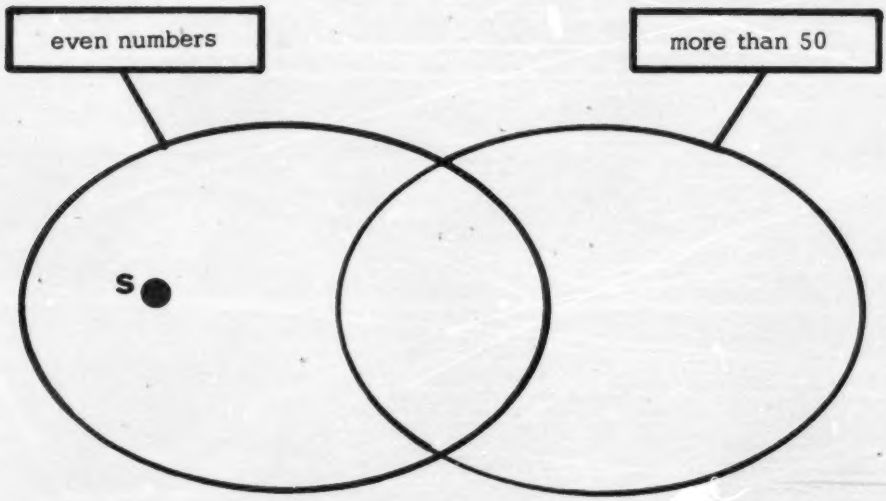
2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72, ...

Explain that " . . ." means that the list goes on forever.

Second clue

T: The second clue is a string picture.

Draw this picture on the board.



T: In this string picture the secret number is here (point to "s").
Is it an even number? (Yes) Is it more than 50? (No)
Which of these numbers could be the secret number?

Each time a number is suggested, label the dot with that number and ask the class if the number is placed correctly. Suppose, for example, a student suggests 72.

T: Could this dot be for 72? (Disagreement)
Is 72 even? (Yes)
Then it must be inside the red string.
Is 72 more than 50? (Yes)
Then is must be inside the blue string.
Where does 72 belong?

Ask a student to point to the correct region.

T: 72 cannot be the secret number.

Erase "72" from the string picture and cross "72" off the list on the board.

If a correct number is suggested, elicit class agreement and circle it on your list. Continue until many of the numbers in the original list are either circled or crossed off.

Point to the three dots.

T: Could the secret number be any of the numbers we didn't write on the board? (No) How do you know?

S: Because the secret number is not more than 50.

Cross off the three dots.

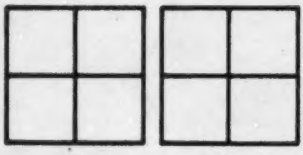
Point to each number not checked and guide your class in deciding if it could be the secret number. When all the numbers have been checked, erase those which have been crossed off.

T: Now we know the secret number is one of these (point to the list of remaining numbers).

EW 2

Third clue

Display two Minicomputer boards.



T: This will be the final clue. At least three (regular) checkers are needed to put the secret number on the Minicomputer. We cannot use less than three checkers to put it on the Minicomputer. When you know which of these numbers is the secret number, tell me with whispers.

Accept several whispers and after most of the students realize that 32 is the secret number, ask a student to say the secret number aloud. Check the other numbers on the Minicomputer by letting students demonstrate that each of them can be put on the Minicomputer using less than three checkers. Reveal that "32" is written on your paper.

EW3 FISHING FOR NUMBERS (Lesson 1)

CAPSULE LESSON SUMMARY

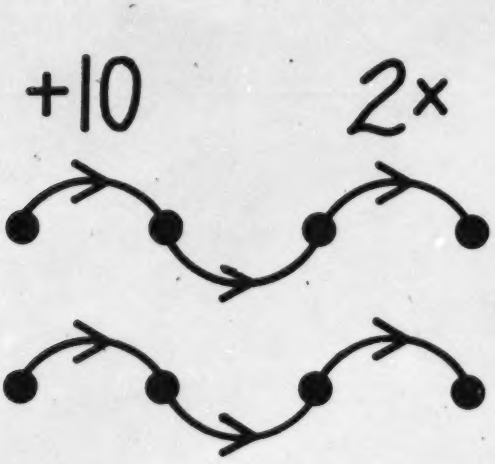
Put the numbers in a string picture in their proper places in an arrow picture. Begin the Workbook FISHING FOR NUMBERS, Part II.

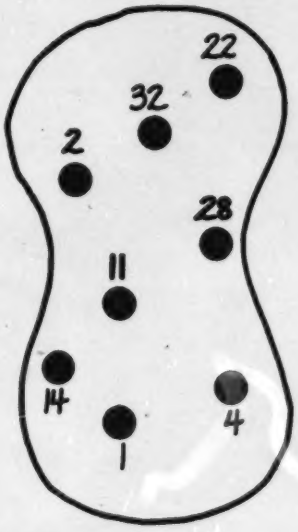
MATERIALS

Teacher: Colored chalk
Student: Workbook FISHING FOR NUMBERS, Part II; pencil

DESCRIPTION OF LESSON

Draw these pictures on the board.

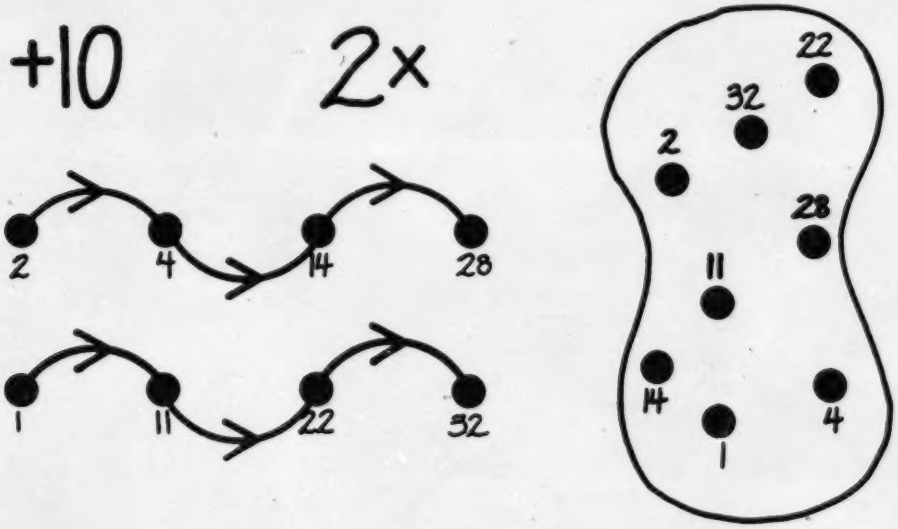




Tell the class that the problem is to put all the numbers shown in the string picture in their proper places in the arrow picture. Ask someone to suggest a number for the first dot of one of the roads. Then pursue that choice and see where it leads. If it leads to an incorrect situation, let the students discover this and discuss why it is wrong and what they should do. A student may suggest that the smallest number must go at the beginning of one of the roads; ask the student

to convince the rest of the class. The students might suggest crossing out the numbers inside the string after they have been placed in the arrow picture. This procedure is questionable since in a trial and error method many mistakes may be made and new starts may be necessary. In the course of solving this problem collectively, the opportunity to discuss various strategies should arise, but allow ample opportunity for students to discover some good strategies themselves.

The completed arrow picture is given below.



Distribute copies of the Workbook FISHING FOR NUMBERS, Part II. Explain to the students that the string picture and the arrow picture are sometimes on the same page and are sometimes on opposite pages. Emphasize that very likely they will need to use their erasers and that they should always make several attempts at labeling the dots in a picture before asking for help.

At the end of the lesson collect the workbooks for use in Lesson EW4.

EW4 FISHING FOR NUMBERS (Lesson 2)

CAPSULE LESSON SUMMARY

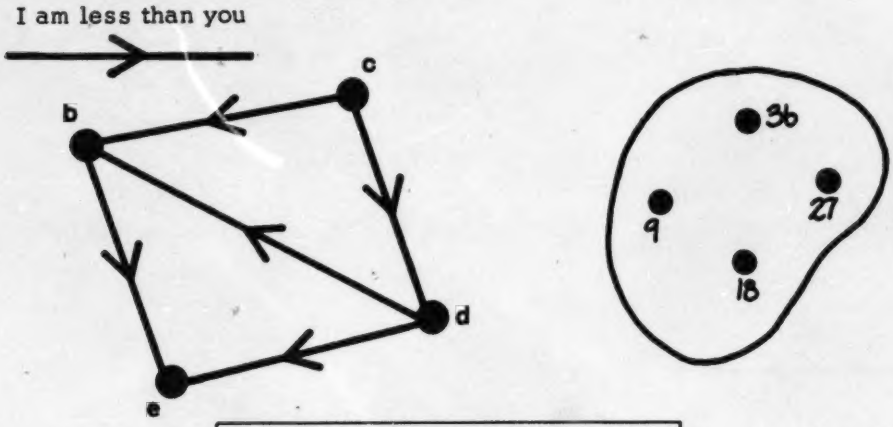
Put the numbers from a string picture in their proper places in an arrow picture which has "I am less than you" arrows. Continue working in the Workbook FISHING FOR NUMBERS. (This is the second of two lessons using this workbook.)

MATERIALS

Teacher: Colored chalk
Student: Workbook FISHING FOR NUMBERS, Part II; pencil

DESCRIPTION OF LESSON

Draw these pictures side by side on the board.



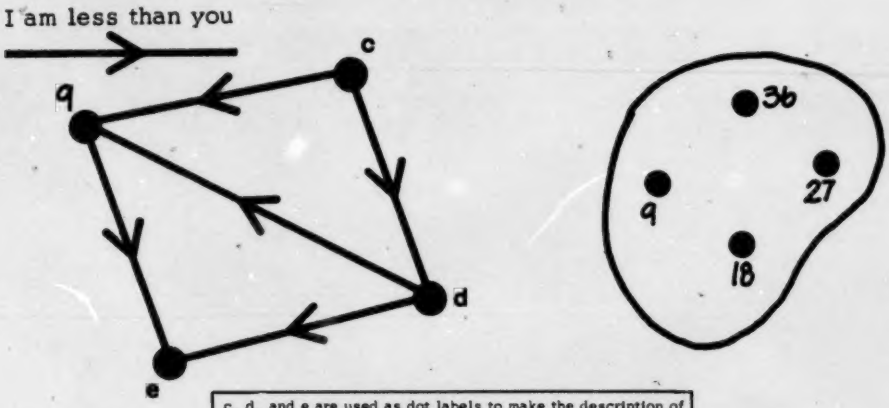
Do not label the dots in the arrow picture. They are here just to make the description of the lesson easier to follow.

T: How many dots are in the arrow picture? (Four)
How many numbers are in the string picture? (Four)
Each of the dots in this arrow picture is for one of the numbers in this string. Each of the numbers has just one dot in this arrow picture.
Who would like to label one of the dots in this arrow picture?

Do not be concerned if the first number chosen is placed incorrectly in the picture, because the subsequent discussion will be of benefit to the class.

EW 4

For example, suppose a student labels b "9".

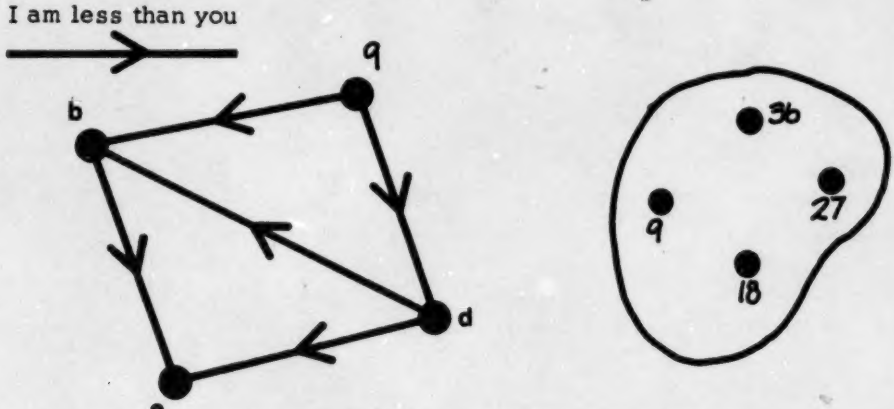


c, d, and e are used as dot labels to make the description of the lesson easier to follow. Do not write them on the board.

T: Could this dot (b) be for "9"? (No)
How do you know?

S: If this number were 9, then this number (at c) would be less than 9, but 9 is the smallest number in this string picture.

Erase "9" from the arrow picture and ask for another volunteer to label one of the dots in the arrow picture. Perhaps a student will label c "9".



b, e, and d are used as dot labels to make the description of the lesson easier to follow. Do not write them on the board.

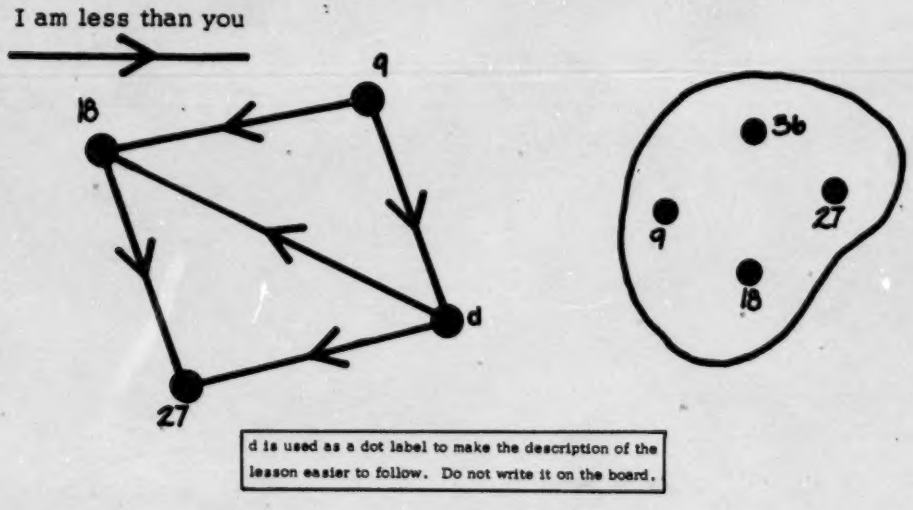
T: Could this dot (c) be for 9?

S: We can't tell yet.

T: How can we find out?

S: We need to label the other dots.

Ask someone to label another dot. Continue this activity until all the dots are labeled or until the class discovers that a labeling is incorrect. Suppose b, c, and e have been labeled as in the next illustration. A sample dialogue follows.



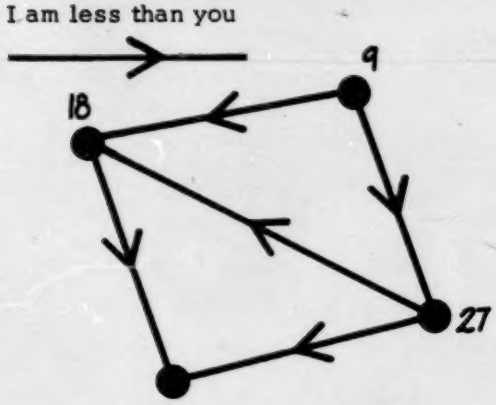
d is used as a dot label to make the description of the lesson easier to follow. Do not write it on the board.

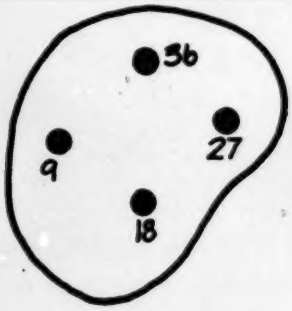
S: This (point to the dot labeled "27") can't be 27 because this number (at d) is less than 27 and we've already used all the numbers less than 27.

S: I think this number (at d) is 27.

EW 4

Erase "27" from the arrow picture and label d "27".





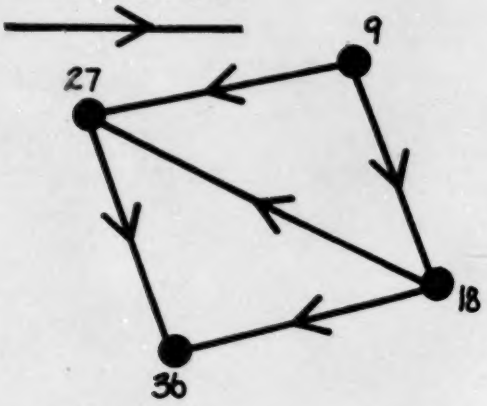
T: Could this dot be for 27?

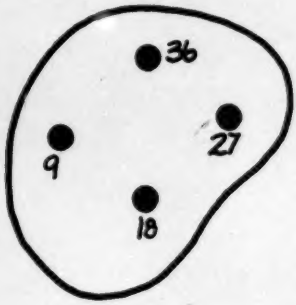
S: No, 27 isn't less than 18.

Erase "27" from the arrow picture.

T: What should we do now?

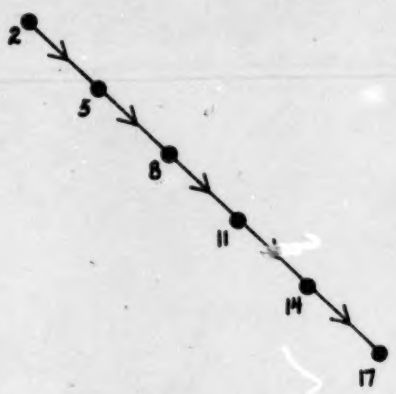
Follow the students' suggestions and continue this activity until all the dots are correctly labeled.



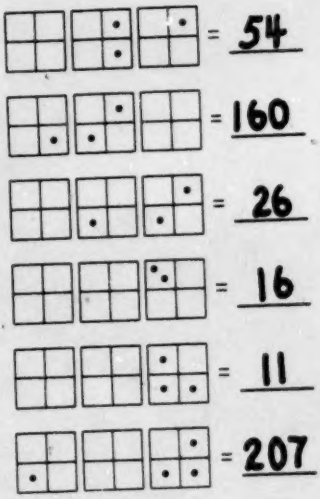


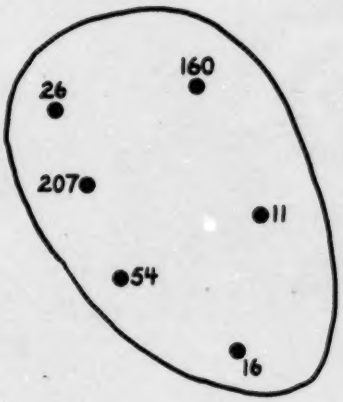
Distribute the students copies of FISHING FOR NUMBERS, Part II. Explain to the students that the string picture and the arrow picture are sometimes on opposite pages and sometimes on the same page. Emphasize that very likely they will need to use their erasers, and that they should always make several attempts at labeling the dots in a picture before asking for help.

At the end of the lesson, collect the workbooks for your review.


+3
2
5
8
11
14
17

8
14
5
17
2
11


= 54
= 160
= 26
= 16
= 11
= 207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26
207
11
54
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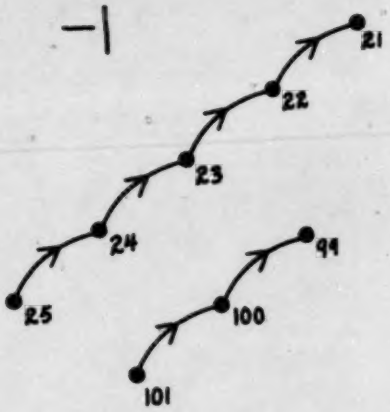
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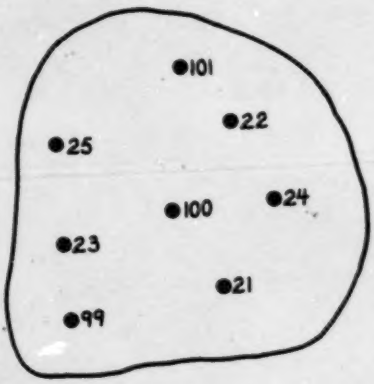
22
23
24
25
99
100
101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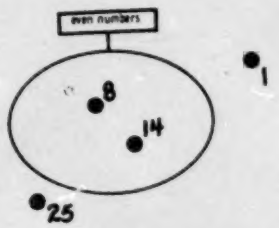
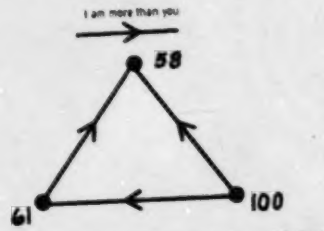
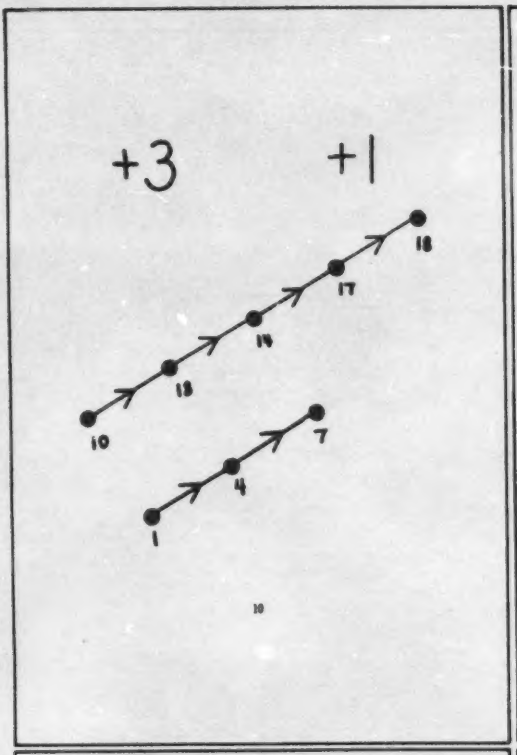
24
100
23
21
99


even numbers
1
8
14
25

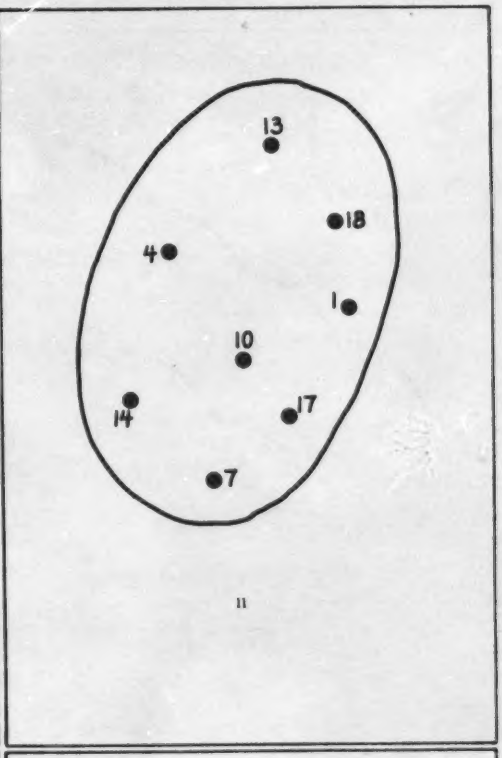
8
1
25
14


I am more than you
58
61
100

100
58
61


+3
+1
18
17
14
13
10
7
4
1

10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18
4
1
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14
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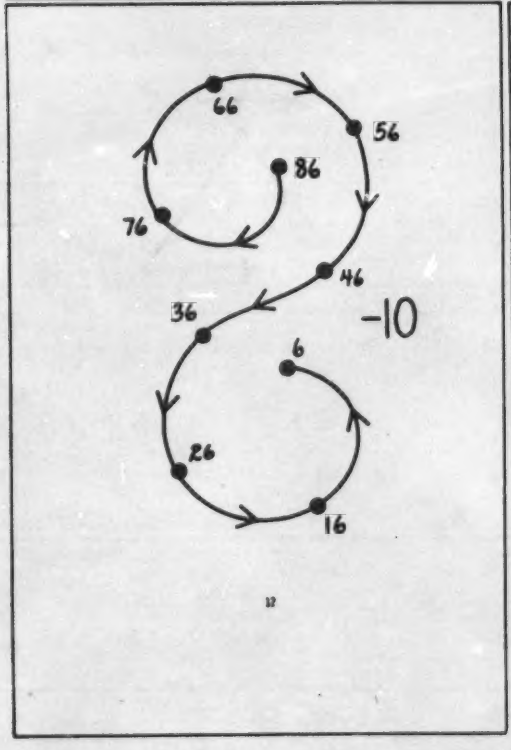
7

11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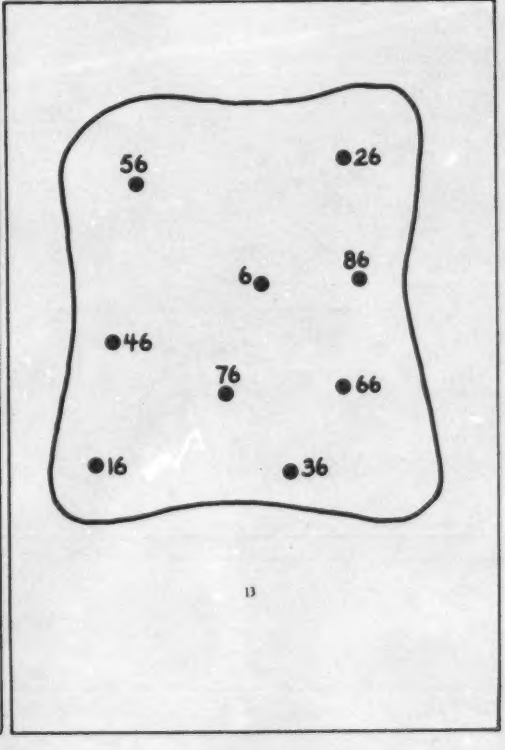
56
86
76
46
36
-10
6
26
16

12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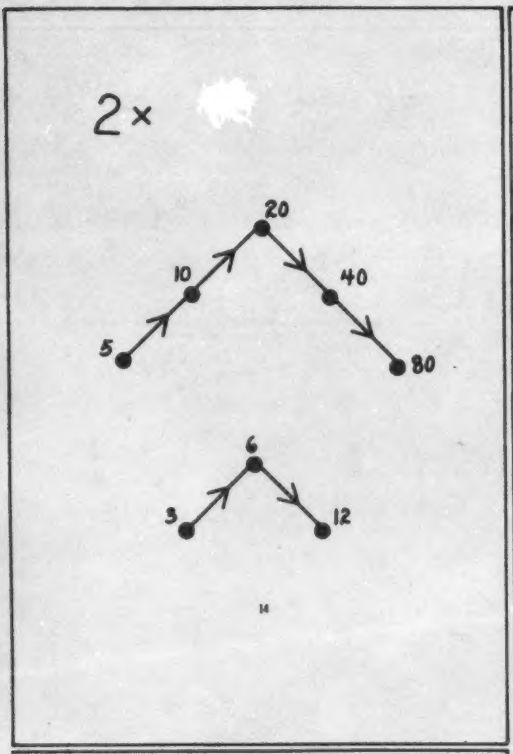
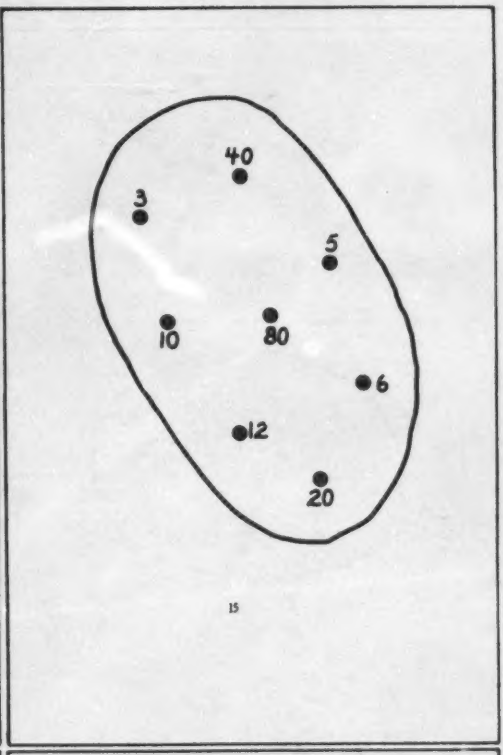

2×
20
10
40
5
80
6
3
12

14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3
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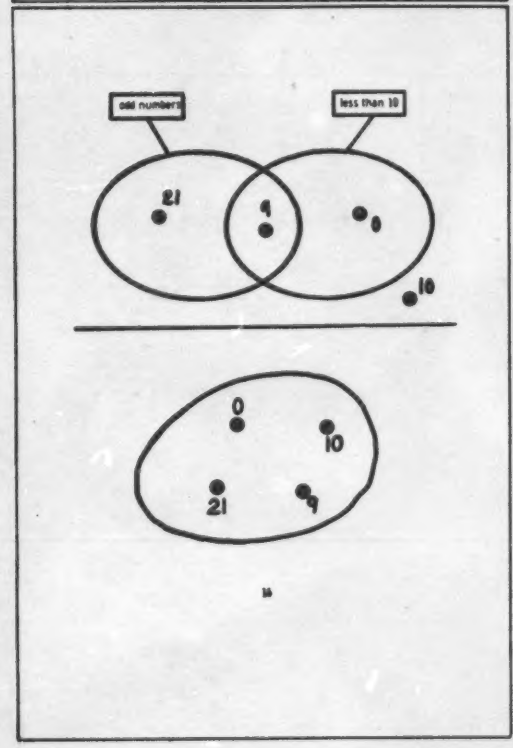
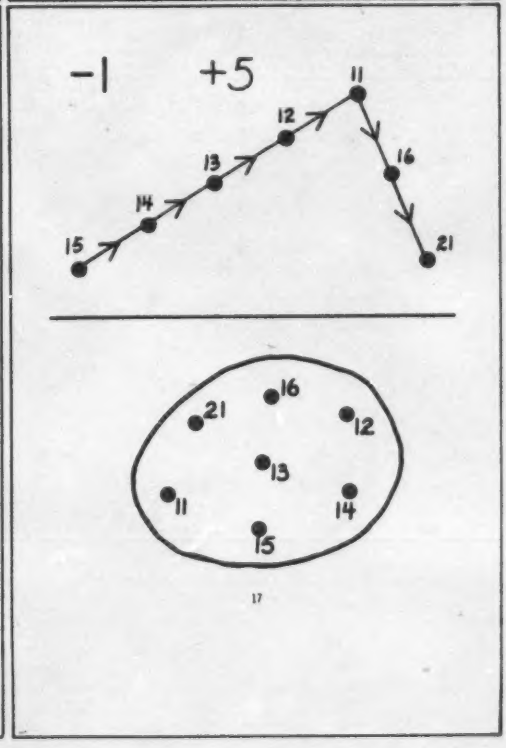
10
80
6
12
20

15


odd numbers
less than 10
21
9
8
10
0
10
21
9

16


-1
+5
11
12
13
14
15
16
21
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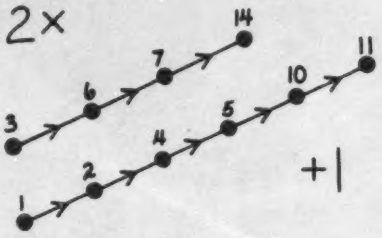
14
15

17


2×
+1
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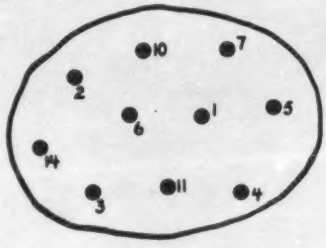
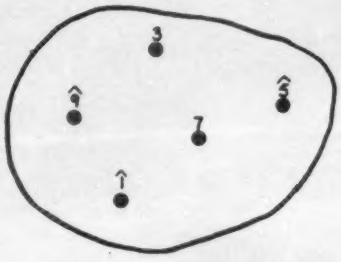
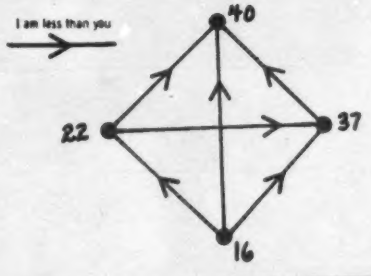

10
7
2
1
5
6
14
11
4
3

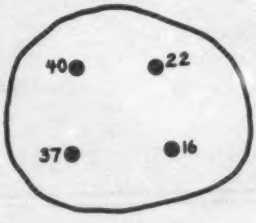
18

+4
9
5
1
3
7

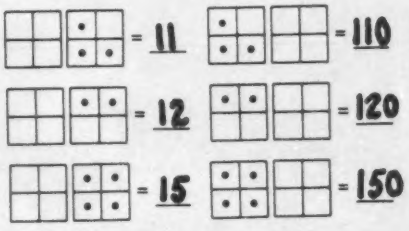

3
9
5
7
1

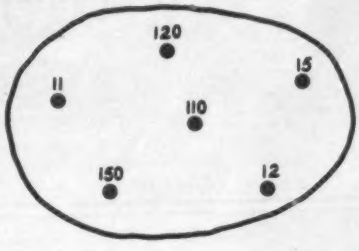
19


I am less than you
40
22
37
16


40
22
37
16

20


= 11
= 110
= 12
= 120
= 15
= 150

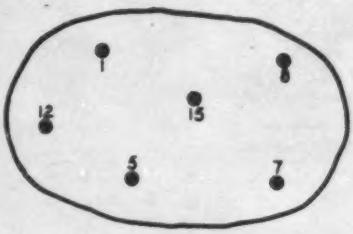

120
15
11
110
150
12

21

12 + 8 = 20

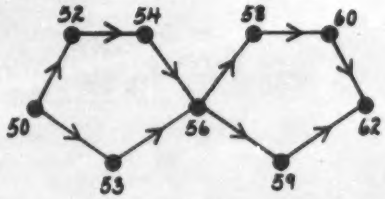
15 + 1 = 16

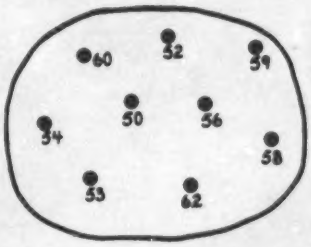
7 + 5 = 12



22

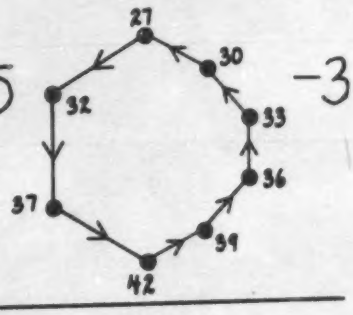
+2 +3

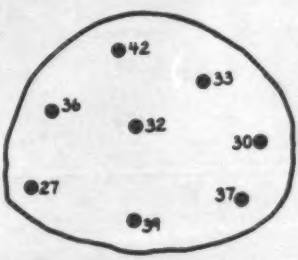




23

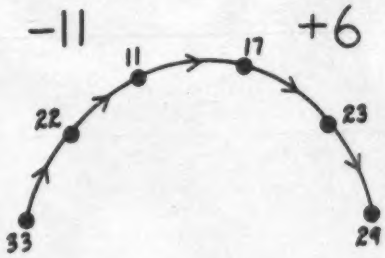
+5 -3

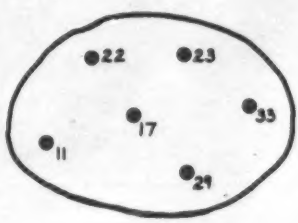




24

-11 +6





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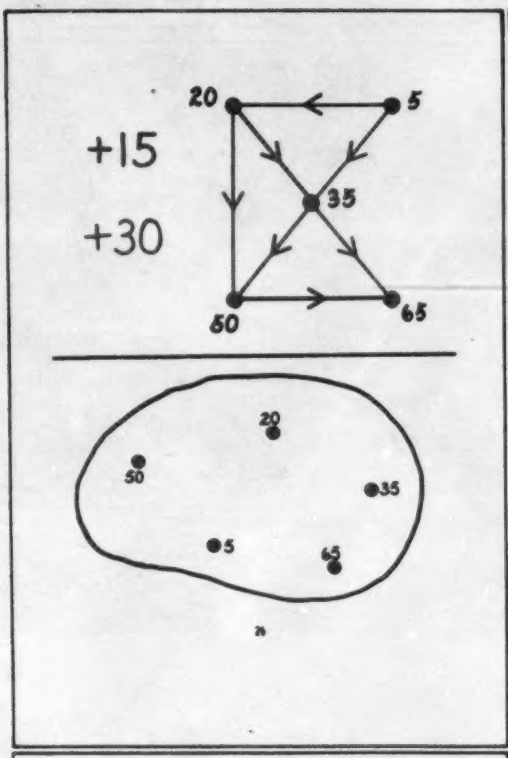
+15
+30
20
5
35
50
65
20
50
35
5
65
26
I am less than you
36 + 19
36-19
36
36
36+19
36-19
27
+6
13
9
3
3
9
0
6
9
6
15
3
9
0
3
28
44
48
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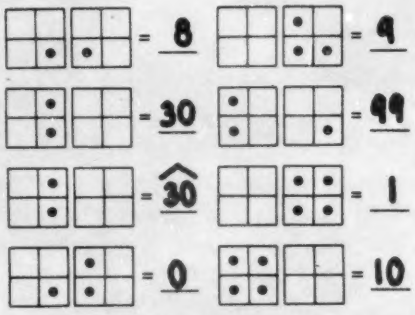
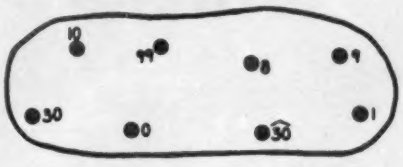
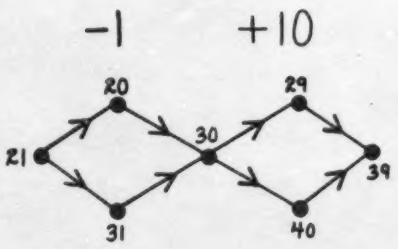
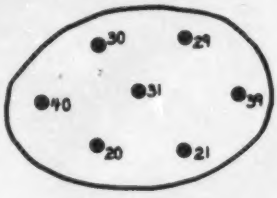
24
11
12
6
3
1/2 x
44
3
48
11
24
6
22
12
29


= 8
= 9
= 30
= 99
= 30
= 1
= 0
= 10

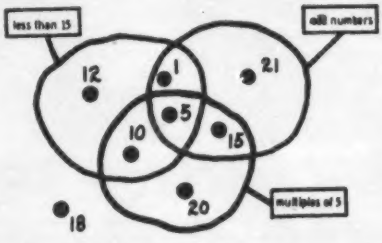

10
99
8
9
30
0
30
1

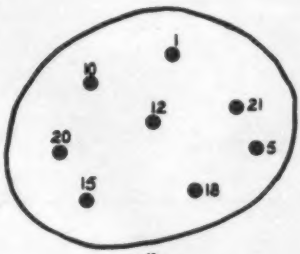
30


-1
+10
20
29
21
30
39
31
40


30
29
40
31
39
20
21

31


less than 15
odd numbers
12
1
21
5
10
15
20
multiples of 5
18


1
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32

EW 5

EW5 Detective Story #3/Eli Tricked by Clarence the Crocodile

CAPSULE LESSON SUMMARY

With an arrow picture, limit the possibilities for a secret number. Give a clue that this secret number is an odd number, and then locate it in a string picture. Present Clarence the Crafty Crocodile who plays a trick on Eli the Elephant. Eli discovers that some of his peanuts are missing. What could Clarence have done in Eli's bag?

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: None

DESCRIPTION OF LESSON

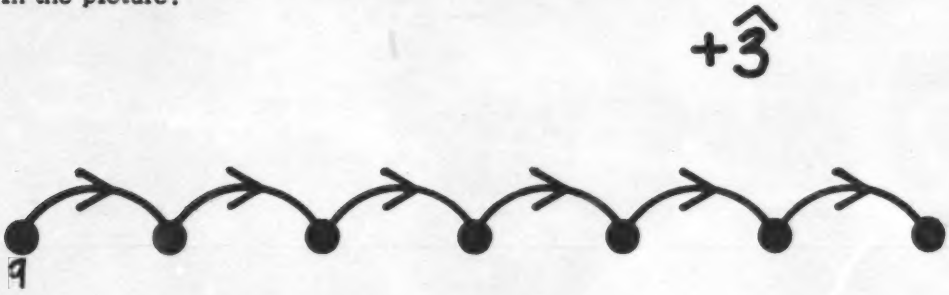
Exercise 1: Detective Story

Write a large "3" on a slip of paper and fold it so that the "3" is hidden.

T: Today you are going to be detectives. You will have several clues to help you discover a secret number.

First clue

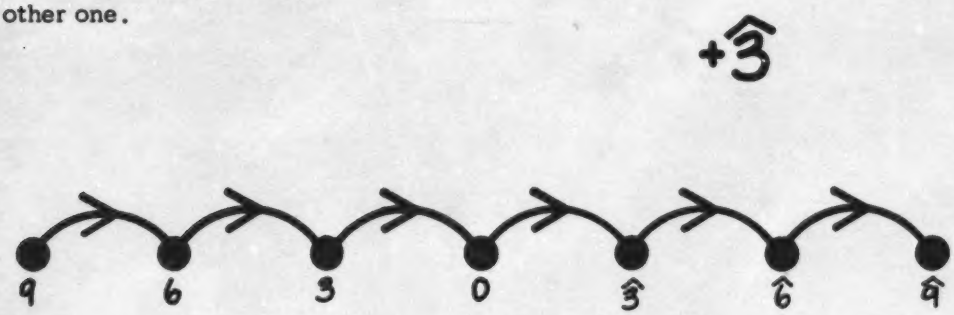
Draw this arrow picture on the board and tell the class that the secret number is in the picture.



Point to "9 and trace the first red arrow as you say, "9 + 3 = . . . ? Nine regular peanuts and three magic peanuts is . . . ?" If your class is not

certain that the second dot from the left is for 6, draw Eli's bag with nine regular peanuts and three magic peanuts. Ask a student to show which peanuts would disappear. Looking at the remaining peanuts, conclude: $9 + \hat{3} = 6$.

You might need to refer frequently to Eli the Elephant to help your students label the remaining dots. Always erase a peanut picture before you draw another one.



After all the dots have been labeled, make a list of the possible numbers on the board . . .

$$9, 6, 3, 0, \hat{3}, \hat{6}, \hat{9}$$

and then erase the arrow picture.

Second clue

T: The second clue is that the secret number is odd. Which of these numbers is odd (point to the list on the board)?

Consider with the class each number in the list and erase any numbers which are not odd.

$$9, 3, \hat{3}, \hat{9}$$

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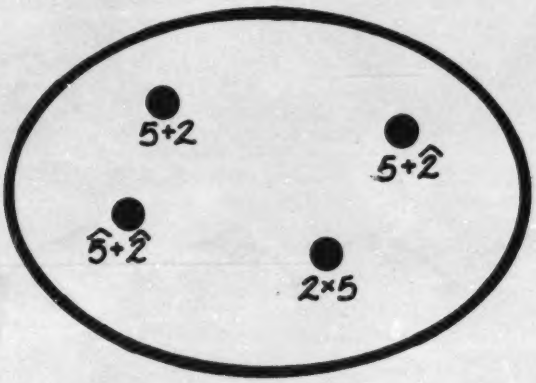
Third clue

T: I will give only one more clue.

Draw this string picture on the board.



T: The secret number is in this string picture. When you know which one is the secret number tell me with whispers.

When most of the students have whispered an answer to you, do each calculatio suggested by the string picture. If necessary, draw peanut pictures to show tha $\hat{5} + \hat{2} = \hat{7}$ and $5 + \hat{2} = 3$.

T: What is the secret number?

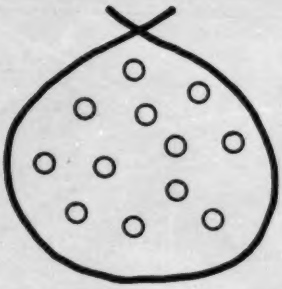
S: 3.

Reveal the "3" on the slip of paper to the class.

Exercise 2: Eli the Elephant

T: One day Eli is walking through the jungle. There are twelve regular peanuts in his bag.

Draw his bag of peanuts on the board.



T: Eli walks a long time; he is very tired and decides to take a nap. As soon as Eli falls asleep, along comes Clarence the Crafty Crocodile. Clarence sees that Eli is asleep and decides that it would be fun to play a trick on Eli. Clarence opens Eli's bag of peanuts and does something. Then he closes Eli's bag quietly and goes away.

Eli feels hungry when he awakes, so he opens his peanut bag. Eli counts his peanuts and cries, "Oh no! Four peanuts are missing." What do you think Clarence the Crafty Crocodile did?

Allow the class to discuss this problem. Two possibilities should emerge: Clarence could have removed four peanuts from Eli's bag, or Clarence could have put four magic peanuts into the bag.

NOTE: There are other more complex possibilities which might be suggested. For example, Clarence could have added two magic peanuts and removed two regular peanuts. Accept such suggestions as correct, but for the purpose of this lesson we are only interested in two possibilities: Clarence removed four regular peanuts, or Clarence added four magic peanuts.

Illustrate each of these situations as it is suggested. Write the appropriate number story under each bag of peanuts. Arrange the board so that the two pictures are side by side. (See the illustration on the next page.)

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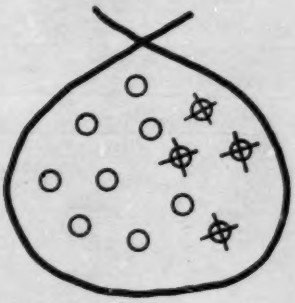
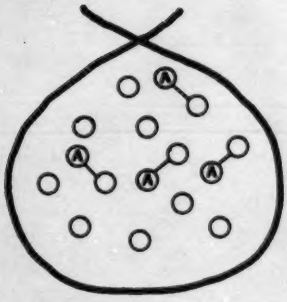


$12 - 4 = 8$



$12 + \hat{4} = 8$

Emphasize that we do not know what Clarence did when he opened Eli's bag, because removing four regular peanuts and adding four magic peanuts have exactly the same effect. Write this as a number story on the board.

$$12 + \hat{4} = 12 - 4$$

T: Another day when Eli is walking in the jungle, he gathers 95 regular peanuts. He becomes very tired and decides to take a nap. Do you know what happens while Eli is sleeping?

S: Clarence the Crafty Crocodile comes by and plays another trick on Eli.

T: Your right! Clarence does come by.

Eli feels hungry when he awakes, so he opens his peanut bag. Eli counts his peanuts and finds that 69 peanuts are missing. What do you think Clarence the Crafty Crocodile did?

S: Clarence could have removed 69 peanuts from Eli's bag.

S: Clarence could have put 69 magic peanuts into Eli's bag.

As each possibility is suggested, write the appropriate number story on the board.

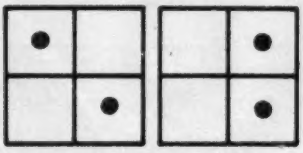
$$95 - 69 = ?$$
$$95 + \widehat{69} = ?$$

T: How are we going to find out how many peanuts Eli has left in his bag?

If someone suggests drawing a picture of Eli's bag, you should accept this as a good, but not a very practical, method because there are so many peanuts you would have to draw. If no one suggests using the Minicomputer, suggest it yourself. Point to the number stories on the board.

T: Which of these problems do you want to do first on the Minicomputer?

Suppose the students want to calculate "95 - 69" first. Invite someone to put 95 on the Minicomputer.



T: Can someone take 69 off the Minicomputer?

S: We do not have checkers on the right squares for 69.

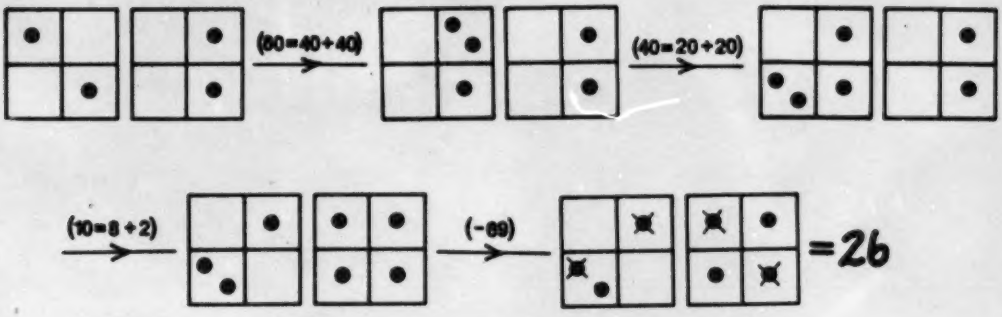
Let someone point to where the checkers for 69 would be.

T: Who can make a backward play which will help us to calculate "95 - 69"?

Ask a student to identify a play before moving checkers. This will help to discourage plays which are not useful. Whenever a play is made which puts a

checker in position for the subtraction, mention this to the class and indicate that they are getting closer to the goal.

Let the students make plays until 69 can be taken off the Minicomputer. A possible sequence of moves is shown below.



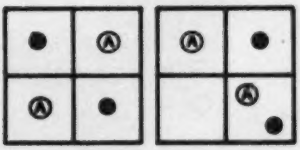
Complete the appropriate number story on the board . . .

$$95 - 69 = 26$$

and remove the checkers from the Minicomputer.

T: Now let's suppose Clarence put 69 magic peanuts into Eli's bag.

Invite a student to put 95 on the Minicomputer and another to put $\widehat{69}$ on the Minicomputer.



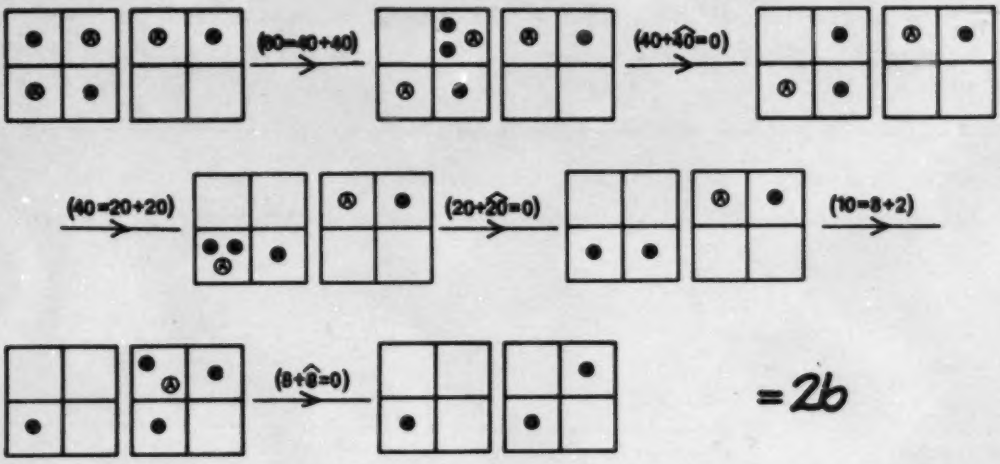
T: We want to get a regular and a negative checker on the same square so that we can remove them. Who can make a play?

Let students make plays until the checkers on the board are all of the same kind.

In this case, they will all be regular checkers.

NOTE: A student might suggest an incorrect play, for example, "8 + $\hat{2}$ = 10". In this case respond "8 + $\hat{2}$ = $\hat{10}$ and 8 + 2 = 10, but 8 + $\hat{2}$ ≠ 10." You might need to illustrate the corresponding situation in Eli's bag of peanuts. For an incorrect play such as "4 + $\hat{4}$ = 8", simply correct the play, "4 + $\hat{4}$ = 0".

One sequence of plays is illustrated below. The two checkers on the 1-square have already been removed.



Complete the number story on the board.

$$95 - 69 = 26$$

$$95 + \hat{69} = 26$$

Emphasize that we do not know what Clarence did when he opened Eli's bag, because removing 69 regular peanuts and adding 69 peanuts have exactly the same effect. Write this as a number story. (See the illustration on the following page.)

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$$95 - 69 = 26$$

$$95 + \widehat{69} = 26$$

$$95 - 69 = 95 + \widehat{69}$$

T: Which calculation did you find easier to do on the Minicomputer?

Accept either response, letting students comment on why they thought one calculation was easier than the other.

Continue this activity with a similar situation where Eli has 83 regular peanuts and later discovers that 57 peanuts are missing. Let the class choose which calculation, either "83 - 59" or "83 + $\widehat{59}$" to do on the Minicomputer. It is not necessary to do both calculations.